The Supermodularity of the Tax Competition Game

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Abstract

Tax competition is often associated with the "race to the bottom:" a decrease in the tax rate of one jurisdiction (country, region or municipality) triggers similar reactions in neighboring jurisdictions. This race may be linked to two properties of the tax competition game: a positive tax spillover and the strategic complementarity of tax rates. Using tools from generalized concavity, more precisely $\mathbf{r}-$concavity, and supermodular games, this paper offers a simple yet unifying perspective on the fundamental forces that shape tax competition. The main results characterize sufficient conditions on the marginal productivity of tax competing jurisdictions to predict a "race to the bottom." These conditions lead us to bind the curvature of the inverse demand for capital of each tax-competing jurisdiction. We deduce several results: at least one pure-strategy Nash equilibrium exists; tax coodination is Pareto improving, but neither communication, nor the coalition of a subgroup of countries does achieve neither tax coordination, nor tax cooperation; for a given stock of capital, any increase in the number of jurisdictions decreases tax rates and tax revenues and improves the net return of capital. Establishing similar sufficient conditions for the supermodularity of the tax competition game with welfare maximizers raises multiple issues. Besides the question of the nature of public spending, we discuss the role of capital by considering an elastic worldwide stock of capital, capital ownership, and the perturbing role of offshore centers.

Keywords: Tax competition; tax coordination; supermodularity; $\mathbf{r}-$concavity.

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1 Introduction

Is tax competition harmful? Can tax coordination be Pareto improving? These questions among others have been addressed in the literature of tax competition initially formalized by Zodrow and Mieszkowski (1986), Wilson (1986) or Wildasin (1988). One of the main conclusions of the literature reviewed by Keen and Konrad (2013) is that international tax competition would trigger a "race to the bottom." In other words, the Nash equilibrium of the standard tax competition game would be characterized by too low tax rates and consequently an underprovision of public goods with respect to the social optimum. This result, which is widely held beyond the academic circle (OECD, 1998, 2013[1]) is far from obvious to establish in a general framework with n *(>2)* asymmetric countries in interaction.

The "race to the bottom" may be viewed as the result of two properties of the tax competition game: a positive tax spillover and the strategic complementarity of tax rates. The first property means that any decrease (increase) in the tax rate of one country reduces (improves) the payoff of the other countries. The second property characterizes the similarity of countries' reaction in any change in the tax rate of one of them: a decrease in one country would induce a similar reply from the other. In contrast, in presence of negative tax spillovers, any decrease in the tax rate of one country improves the payoff of the others. Tax competition would then have a positive impact on countries' payoffs. Such a view is in line with the *Public Choice* school, which considers tax competition as a way to tame the *Leviathan* (see Brennan and Buchanan, 1980). If tax rates are strategic substitutes, any change of the tax rate in one country would imply an opposite reaction by the others and neither a "race to the bottom" nor a "race to the top" may take place, the need for some tax coordination becoming dubious.

On the empirical side, a significant number of works, reviewed in Leibrecht and Hochgatterer (2012), Devereux and Loretz (2013) or Costa-Font, De-Albuquerque, and Doucouliagos (2014), focus on the existence of tax competition and its nature. A large body of this literature establishes the existence of positive slopes of the tax reaction function[2] or equivalently the strategic complementarity of tax rates. However, it is worth to remark that some recent analyses

[1] The May 2016 World Bank Conference: "Winning the Tax Wars: Global Solutions for Developing Countries" stresses also how the "race to the bottom" concerns developing countries too and undermines their domestic resource mobilization efforts.

[2] For Devereux and Loretz (2013) this is the "most important empirically testable hypothesis" in the literature on tax competition. Costa-Font et al. (2014) develop a meta-regression analysis covering 65 empirical studies on horizontal tax competition. They confirm the existence of positive reaction functions for all countries with significant differences among them, tax competition being less intense for Germany, Italy, and Switzerland.

(Chirinko and Wilson, 2007; Parchet, 2014) display downward sloped reaction functions (respectively, among US states and Swiss municipalities) leaving the question of the nature of tax competition open for further empirical investigations. The degree of tax spillovers has been investigated in the recent study from the International Monetary Fund (IMF, 2014), which establishes positive tax spillovers based on panel data of corporate income tax for 103 countries for the period 1980-2013. This result may be also related to the literature on tax planning activities by multinational companies initiated by Hines and Rice (1994) and reviewed by Dharmapala (2014). These works appreciate the impact of tax rates differential between jurisdictions on income shifting between affiliates and parents of multinational companies. A decrease in the tax rate of one jurisdiction triggers some income shifting towards this jurisdiction, which corresponds to a positive tax spillover at the macroeconomic level.

Positive tax spillovers and strategic complementarities of tax rates are critical for a "race to the bottom." These properties are often implicitly assumed in the literature or derived from the analytical specification of the production function used by the authors. We study here sufficient conditions to obtain plain and strategic complementarities of tax rates with general production function, which may differ among jurisdictions. If the first property is immediate in our framework where countries maximize their tax revenue, the second one is more delicate to establish. We follow the standard approach as originated by Zodrow and Mieszkowski (1986) and Wilson (1986) to apprehend tax competition: capital is perfectly mobile; its net return is then equal accross countries. We consider n countries, which differ by their respective production function. Countries choose simultaneously their tax policy, here their respective tax rate, looking for maximizing their tax revenue under the constraint of the perfect mobility of capital. While Zodrow and Mieszkowski (1986), Wilson (1986) and Wildasin (1988) consider welfare maximizer, we restrict our analysis to tax revenue maximizer. Our approach is very simple given countries' actual tax systems. However, first we can interpret tax rates as average effective tax rates, which encompasses statutory tax rates, tax base's definitions, and even tax law enforcement's dimension. Second considering tax revenue maximization allows us to focus on the building block of the tax competition game and more broadly of fiscal competition. Establishing general sufficient conditions for the supermodularity of the tax competition game in the presence of welfare maximizers raises multiple issues: (i) the nature of public goods and their degree of substitutability or complementarity with respect to private consumption; (ii) the weight of capital owners in the welfare function, the distribution of capital, and the intra- and interjurisdiction redistributive impact of tax competition. Finally, our analysis may be viewed as a preliminary step in the understanding of tax systems' competition by introducing a powerful tool: supermodularity to deal with the mutlidimensionality of tax systems (see Slemrod and Gillitzer 2014).

In the tax competition game, we study here, equilibrium tax rates are implicitly defined through the first order condition of the constrained maximization program of each country. The marginal production function plays a crucial role since it corresponds to the inverse demand for capital. And the curvature of the demand function is determinant for the supermodularity of tax competition as it is in oligopoly theory. Given that this demand function is not directly defined in our framework in contrast to oligopoly theory, we will have to use a concept of generalized concavity (see Avriel, Diewert, Schaible, and Zang 1988), more specifically this of $\mathbf{r}-$concavity introduced in economics by Aumann (1975). Then, we identify sufficient conditions for the supermodularity of the tax competition. Supermodular games, which have been mainly applied in industrial organization[3] display several nice properties: first, they encompass many analytical specifications, allowing appreciation of the robustness of the results; second, the existence of at least one pure-strategy Nash equilibrium is immediate, and many solution concepts yield the same prediction; finally, these games tend to be analytically appealing by significantly simplifying the analysis.[4] These three qualities are particularly relevant in the context of tax competition, where the formalization of the problem differs among authors and the existence of a Nash equilibrium remains an issue.

We establish that the $\mathbf{r}-$concavity of marginal production function (with $\mathbf{r} \leqslant 1$) is a sufficient conditions for the supermodularity of the tax competition game, when countries maximize their tax revenue and under standard assumptions on the production function. These conditions allow us to bind the curvature of the marginal production function. Quadratic production function respects all these conditions, while an additional condition relating the stock of capital to the productivity parameters is necessary for Cobb-Douglas production function. Our set of sufficient conditions may appear stringent, but the tax competition game brings into play two opposite effects resulting from any variation in the tax rate of one country on the other countries' tax

[3] See Topkis (1998), Vives (1999), Amir (2005) and Vives (2005).

[4] Vives (2005) wrote:

> "The beauty of the approach is not its complexity but rather how much it simplifies the analysis and clarifies results. In fact, even the basic tools [of the theory of supermodular games] are not fully exploited by economists in current research."

policy: (i) an increase in the tax rate of country i involves a decrease in the worldwide net return of capital, which allows the other countries to increase their own tax rate *ceteris paribus*; (ii) this increase triggers also an outflow of capital from country i to the others, which improves the tax base of the latter, but decreases their marginal productivity and induces them to reduce their own tax rates.

Our analysis contributes to the literature in twofold. First, it establishes sufficient conditions for the strategic complementarity of tax rates. In this regard, it provides some theoretical backgrounds to the seminal works of Zodrow and Mieszkowski (1986) and Wilson (1986). In particular, it complements Laussel and Le Breton (1998), who address the issue of the existence and uniqueness of Nash equilibrium in tax competition.[5] We also establish a sufficient condition for the strategic complementarity of tax rates with Cobb-Douglas production functions. Second, it applies several results of supermodular games to tax competition. These results hold for any payoff function especially welfare function, which displays the property of the strategic complementarity of tax rates. From the supermodularity of the tax competition game we deduce the existence of at least one pure-strategy Nash equilibrium. We then distinguish tax coordination from tax cooperation. Following the literature on macroeconomic coordination failures, we consider that there is a tax coordination problem, when countries fail to coordinate on the Pareto dominant Nash equilibrium, while tax cooperation consists of reaching a Pareto superior outcome, which does not have to be a Nash equilibrium of the initial tax competition game. With these definitions and given the property of positive tax spillovers we establish that tax coordination is unambiguously Pareto improving. We also show that neither cheap talk (costless communication) nor coalition can solve, respectively, the tax coordination or the tax cooperation issue. Finally, we highlight that, the stock of available capital being constant, any increase in the number of tax-competing countries reduces tax rates and tax revenue, and improves the net return of capital if and only if tax rates are strategic complements.[6] All these results hold with welfare maximizers or in other forms of tax competition (e.g., commodity or excise tax competition) as long as the tax competition game is supermodular.[7]

Before concluding, we highlight the role of capital supply, which jeopardizes seriously the con-

[5]In a symmetric two-country model and in absence of capital owners Laussel and Le Breton (1998) emphasize that the tax competition game is neither concave, nor supermodular. By restricting our analysis to capital tax revenue maximizer, we circumvent some issues raised by these authors.

[6]In contrast, strategic substitutability of tax rates means an opposite relationship between the number of competing countries and the level of tax rates.

[7]For instance, it is straightforward to establish the supermodularity of the commodity tax competition proposed by Kanbur and Keen (1993) despite that reaction functions are not always differentiable there.

ventional view of an harmful tax competition even when countries are looking for maximizing tax revenue only. First, we relax the implicit assumption of an inelastic worldwide stock of capital by considering saving decisions. This induces a positive relationship between interest rate and the total stock of capital. The supermodularity of the tax competition game may still hold if the convexity of the saving function remains moderate. Second, capital ownership, its distribution, and its potential concentration in some countries lead us to consider a new kind of player in the tax competition game: offshore finance centers or tax havens. Characterized by a zero capital tax rate and no real economic activity,[8] tax havens are singular players displaying negative tax spillovers and eventually strategic substitutability of their tax rates. They may modify drastically the international tax competition, which is not supermodular anymore.

The paper is structured as follows: section 2 is a preamble introducing some results regarding generalized concavity; section 3 presents the tax competition game and sufficient conditions for its supermodularity; in section 4 we deduce some consequences of the supermodularity of the tax competition game in particular in terms of tax coordination and cooperation; section 5 discusses the role of capital on the nature of tax competition; section 6 concludes.

2 Preamble: Generalized concavity and $\mathbf{r}-$concavity

As a preamble, we present the concepts of generalized concavity and $\mathbf{r}-$concavity following Avriel et al. (1988).

Definition 1. Let h be a real-valued continuous function defined on the convex set $C \subset \mathbb{R}^n$, and denote by $I_h(C)$ the range of h; that is, the image of C under h. The function h is said to be $G-$concave ($G-$convex) if there exists a continuous real-valued increasing function G defined on $I_h(C)$, such that $G(h(x))$ is concave (convex) over C.

We will use a subset of $G-$concave functions by considering $\mathbf{r}-$concave functions. Balogh and Ewerhart (2015) describes the origin of this function, which was independently defined by Martos (1966) and by Avriel (1972). The former was looking for a generalization of concavity allowing the transition between concave and quasi-concave functions. Zhao, Wang, and Coladas Uria (2010) reviews some characterics of $\mathbf{r}-$convex functions. We have the following definition:

Definition 2. Let h be a real-valued nonnegative continuous function defined on the convex

[8] We do not consider here secrecy, which is another characteristic of many tax havens and induces tax evasion.

set $C \subset \mathbb{R}^{+n}$. The function h is **r**-concave if there exists a nonnegative real number r such that

$$\forall \lambda \in [0,1]\,,\; h(\lambda x_1 + (1-\lambda)x_2) \geqslant \begin{cases} -log\left\{\lambda e^{-rh(x_1)} + (1-\lambda)\,e^{-rh(x_2)}\right\}^{1/r} & if\ r \neq 0 \\ \lambda h(x_1) + (1-\lambda)\,h(x_2) & if\ r = 0 \end{cases}$$

The notion of **r**−concavity is closely related to this of ρ-concavity (see Lemma 1 in Balogh and Ewerhart, 2015).[9] Considering **r**−concave functions in $\mathbb{R}$, which are twice continuously differentiable, we have the following definition (see Proposition 8.9, Page 240 in Avriel et al. 1988):

Definition 3. Let h be a real-valued nonnegative continuous function twice continuously differentiable on the convex set $C \subset \mathbb{R}^{+}$. The function h is **r**-concave on C for some $\mathbf{r} > 0$ if and only if $-Exp\left(-\mathbf{r}h(x)\right)$ is concave or equivalently if and only if

$$h''(x) - \mathbf{r}\left(h'(x)\right)^2 \leqslant 0. \tag{1}$$

Strict **r**−concavity is similarly defined with strict inequality and the notion of **r**−convexity with the opposite inequality. The case of standard concavity is equivalent to $\mathbf{r} = 0$. Aumann (1975) and Caplin and Nalebuff (1991) introduced respectively **r**−concavity and ρ-concavity to the economics literature. Anderson and Renault (2003) use the latter to determine efficiency and surplus bounds in the Cournot oligopoly. Ewerhart (2014) develops (α,β)-biconcavity, which corresponds to an exponential transformation of price and quantity in the Cournot model. He deduces simple conditions on α and β for the existence and the uniqueness of a pure-strategy Nash equilibrium. Bagnoli and Bergstrom (2005) review the applications of log-concavity in economics, which is a particular case of ρ-concavity.

We will use a result derived from Proposition (8.19) in Avriel et al. (1988) and applied to real-valued nonnegative continuous function twice continuously differentiable:

Lemma 1. *Let h be a real-valued nonnegative continuous function twice continuously differentiable on the convex set $C \subset \mathbb{R}^{+}$. Then h is $\bar{\mathbf{r}}$−concave on C with $\bar{\mathbf{r}} = max\left(0, \mathbf{r}^*\right)$ where $\mathbf{r}^*$ is*

[9] A real-valued function h is ρ-concave if there exists a real number ρ, such that

$$\forall \lambda \in [0,1]\,,\; h(\lambda x_1 + (1-\lambda)x_2) \geqslant \begin{cases} \lambda\left(h(x_1)\right)^{\rho} + (1-\lambda)\left(h(x_2)\right)^{1-\rho} & if\ \rho \neq 0 \\ \left(h(x_1)\right)^{\lambda}\left(h(x_2)\right)^{1-\lambda} & if\ \rho = 0 \end{cases}$$

Lemma 1 in Balogh and Ewerhart (2015) states that for any $\mathbf{r} = \rho \in [-\infty, \infty]$, a real-valued function f is **r**−concave if and only if $g = Exp[f]$ is ρ−concave.

the supremal value defined as

$$\mathbf{r}^* \equiv \underset{x \in C}{sup} \left\{ \frac{h''(x)}{[h'(x)]^2} \right\} < \infty.$$

3 The tax competition game

The basic framework of tax competition ascribed to Zodrow and Mieszkowski (1986) and Wilson (1986), is a one-period model featuring a single good produced by two factors: labor, which is immobile across countries, and capital (k_i), which is perfectly mobile. The government of each jurisdiction chooses its tax rate on capital (the mobile production factor) to maximize a welfare function. In contrast to these seminal articles and a large part of the literature on tax competition, we consider tax revenue maximizers in this section. We will discuss potential extensions in section 5. However, we emphasize first that tax revenue is the cornerstone of any model of tax competition; second, establishing the supermodularity of this game may be viewed as a preliminary step to a broader approach; finally, a large number of empirical works on tax competition consider countries' tax revenue as the dependent variable (see Leibrecht and Hochgatterer, 2012 and Devereux and Loretz, 2013).

We study the following tax competition game, denoted by $\Gamma \equiv (S_i, R_i; i \in \mathbf{N})$, where S_i is the strategy set of country i, R_i is its payoff function, and n is the number of interacting countries. Each country maximizes simultaneously its tax revenue (R_i) with respect to its own tax rate, denoted by t_i, under the constraint of capital's perfect mobility. The strategy set of each country (S_i) is identical and corresponds to the interval $S_i \equiv [0, 1]$.[10] Country i's payoff function is given by[11]

$$R_i(t) \equiv t_i k_i(t), \tag{2}$$

where t is the vector of tax rates ($t \equiv (t_1, .., t_N)$).

The production function in country i is denoted by $f_i(k_i)$ and differs among countries.[12] We assume the following:

Assumption (1):

$$\forall k_i > 0, \quad f_i'(k_i) \geqslant 0 > f_i''(k_i). \tag{3}$$

[10] We consider that tax rates cannot be negative.

[11] The concavity of $R_i(t)$ is established in Appendix A.1.

[12] Fixed factors as explicit arguments of the production function are suppressed.

Assumption (2):

$$\forall k_i > 0, \quad f_i''' (k_i) \geqslant 0. \tag{4}$$

The production function of each country is increasing and concave in capital. The nonnegativity of the third derivative of the production function is often assumed in the tax competition literature. It shapes the curvature of capital demand in each country and their respective tax policy.

Following the literature on tax competition, we assume that firms behave competitively in each country. Capital is priced at its net marginal productivity: $f'(k_i) - t_i = r$, where r depends on t. Capital being perfectly mobile across countries and the total stock of capital being fixed equal to $\overline{k}$, the market-clearing conditions is given by:

$$\begin{cases} f_i'(k_i) - t_i = r, \quad \forall i \in \{1, ..., n\} \\ \sum\limits_{i=1}^{n} k_i = \overline{k} \end{cases} \tag{5}$$

We add the following assumption, which ensures the nonnegativity of the net return of capital (r) and rules out corner solution ($k_i = 0$) and capital free disposal ($\sum\limits_{i=1}^{n} k_i < \overline{k}$). We have:[13]

Assumption (3):

$$\forall i \in \{1, ..., N\}, \quad f_i'\left(\bar{k}\right) > 1. \tag{6}$$

From (5) and applying the Implicit Function Theorem, we deduce some standard results, already established in the literature (Wildasin, 1988; Keen and Konrad, 2013):

$$\frac{\partial r}{\partial t_i} = -\frac{\frac{1}{f_i''(k_i)}}{\sum\limits_{l=1}^{n} \frac{1}{f_l''(k_l)}} \in \left]-1, 0\right[, \tag{7}$$

and

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f_i''(k_i)}\left(1 + \frac{\partial r}{\partial t_i}\right) < 0 \ \ and \ \ \frac{\partial k_j}{\partial t_i} = \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i} > 0. \tag{8}$$

The net return of capital (r) is decreasing and convex in the tax rate of each country. The convexity of r derives from Assumption 2 (see Appendix A.1). The demand of capital in country i is a decreasing and concave[14] function in the tax rate of this country (t_i) and an increasing function in the tax rate of the other country (t_j).

[13]Indeed, by the following assumption we get: $\forall i \in \{1, ..., N\}, \quad r = f_i'(k_i) - t_i > f_i'\left(\overline{k}\right) - 1 > 0$.

[14]The concavity derives from Assumption 2 (see Appendix A.1).

An important property of the game (Γ) is the positive tax spillover or equivalently the plain complementarity of tax rates following the taxonomy proposed by Eaton (2004).[15] In other words, the payoff function is increasing (nondecreasing) in the strategic variables of the other players:

$$\frac{\partial R_i(t)}{\partial t_j} = t_i \frac{\partial k_i}{\partial t_j} = t_i \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} > 0. \tag{9}$$

This property reflects the *tax base effect*: any increase in the tax rate in country j reduces the net return of capital in this country and drives out capital from this country into country i; this flow broadens the capital tax base of country i and increases its tax revenue.

Applying Definition 3, we consider an additional assumption, which are critical to establish the supermodularity of the tax competition game:

Assumption 4: The marginal production function of country i: $f_i'(.)$ is $\mathbf{r}-$concave or equivalently

$$f_i'''(k_i) - \mathbf{r}\left(f_i''(k_i)\right)^2 \leqslant 0. \tag{10}$$

As Wildasin (1988) highlighted, the marginal production function is also the inverse demand function for capital in country i. From Assumptions 2 and given the market clearing condition (5), we can deduce that the demand for capital in country i, denoted by $d_i(.)$, is decreasing and convex with respect to the gross return of capital in country i: $r + t_i$. Indeed, we have: $d_i(r+t_i) = f_i'^{-1}(r+t_i)$, $d_i'(r+t_i) = \frac{1}{f_i''(k_i)} < 0$, and $d_i''(r+t_i) = -\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^3} \geqslant 0$. Assumption (4) allows us to bound the convexity of $d_i(.)$ and consequently to shape best-reply functions.

Given the unidimensionality of the strategy set, the supermodularity of the tax competition game derives from the strategic complementarity of tax rates as defined by Bulow, Geanakoplos, and Klemperer (1985): any increase (decrease) in the tax rate of one country induces a similar variation in the tax rate of the other country. Our main result is to provide sufficient conditions on the production function, which involve the supermodularity of the tax competition game.

Proposition 1. *Under Assumptions 1 to 4, the tax competition game (Γ) is supermodular if $\mathbf{r} \leqslant 1$.*

Proof. See Appendix A.2. □

The $\mathbf{r}-$concavity of the marginal production function with $\mathbf{r} \leqslant 1$ is a sufficient condition for

[15] Plain complementarity is equivalent to positive spillovers, while plain substitutability corresponds to negative spillovers. We follow Eaton's terminology for its clarity and its "complementarity" with the notions of strategic complementarity and substitutability used below.

the strategic complementarity of tax rates and consequently for the supermodularity of the tax competition game. Assumptions 2 and 4 bind the curvature of the jurisdiction's demand for capital. In the tax competition game Γ, any variation in country i's tax rate triggers two opposite effects on the tax rate of the other countries. On one side, an increase in country i's tax rate (t_i) reduces the worldwide net return of capital (r) modifying the market clearing conditions and making the constraint of capital perfect mobility less demanding for all the competing countries. On the other side, this variation induces the tax base effect, which increases the stock of capital in country j and consequently decreases its gross return of capital (due to the concavity of the production function). From (5) we have $f_j'(k_j) - r = t_j$ and we note that the two previous effects resulting from an increase in t_i have an opposite impact on t_j. If the first effect dominates, tax rates are strategic complements. Otherwise, they are strategic substitutes.

Proposition 1 is close to some results established in the industrial organization literature. For instance, studying Bertrand duopoly, Amir (1996) establishes that the log-concavity of the demand function is a sufficient condition for the supermodularity of this game. Anderson and Renault (2003) apply $\rho-$concavity and $\rho-$convexity to demand function to establish some bounds on the ratios of deadweight loss and consumer surplus to producer surplus in Cournot competition. Our analysis consists mainly in establishing some limits on the shape of countries' demand for capital.

The quadratic production function often used in the tax competition literature fulfills all the sufficient conditions for the supermodularity of the tax competition game Γ: $f_i(k_i) = k_i(a_i - b_i k_i)$, with $a_i, b_i > 0$, $f_i'(k_i) = a_i - 2b_i k_i \geqslant 0$ under the additional assumption that $\forall i \in N$, $\bar{k} \leqslant a_i/2b_i$, $f_i''(k_i) = -2b_i < 0$, $f_i'''(k_i) = 0 \geqslant 0$, $(-Exp(-\mathbf{r} f_i'(k_i)))'' = -\mathbf{r}^2 b_i Exp(-\mathbf{r} f_i'(k_i)) \leqslant 0$ for $0 < \mathbf{r} \leqslant 1$. Considering the Cobb-Douglas production function widely used in macroeconomics we establish the following Corollary:

Corollary 1. *Tax rates are strategic complements if countries maximize their tax revenue, production functions are Cobb-Douglas type ($f_i(k_i) = A_i k_i^{\alpha_i}$ with $A_i > 0$ and $0 < \alpha_i < 1$), and the following condition holds:*

$$\bar{k}^{1-\underline{\alpha}} \leqslant \underline{A} \frac{\bar{\alpha}(1-\underline{\alpha})}{2-\underline{\alpha}}, \tag{11}$$

where $\underline{A} \equiv \min_{i \in N} \{A_i\}$, $\underline{\alpha} \equiv \min_{i \in N} \{\alpha_i\}$, *and* $\bar{\alpha} \equiv \max_{i \in N} \{\alpha_i\}$.

Proof. See Appendix A.3. □

The necessary condition (11) relates the stock of capital with the extremal values of countries' production functions. This condition is more stringent than Assumption 3 for the Cobb-Douglas function.

The existence of Nash equilibrium is an immediate consequence of the supermodularity of the studied game. We have:

Corollary 2. *Under Assumptions 1 to 4, there is always a pure-strategy Nash equilibrium of the tax competition game* (Γ)*.*

Proof. See Topkis (1998), Theorem 4.2.1, page 181 or Vives (2005), Theorem 2.5, page 33. □

The existence of the Nash equilibrium follows directly from the analysis of Topkis (1998). Several authors have studied this issue in the tax competition context. For instance, Bucovetsky (1991), Wildasin (1991) or Wilson (1991), specified their objective functions in such a way that countries' best replies are linear and cross once, which implies the existence and the uniqueness of the Nash equilibrium. Laussel and Le Breton (1998) establish the existence of the Nash equilibrium in a symmetric two-country framework, but still under some restrictive assumptions: (*i*) the convexity of the marginal production function, (*ii*) the linearity of the objective functions in public and private consumption, and (*iii*) the absence of capital owners in these functions. Their studied payoff function, which we denote by $W_i(t)$, corresponds to the sum of the fixed factors income and the capital tax revenue: $W_i(t) = f_i(k_i) - k_i f_i'(k_i) + R_i(t)$. Our approach dos not allow to establish the supermodularity of this function.[16] Some recent papers (e.g., Bayindir-Upmann and Ziad, 2005, or Taugourdeau and Ziad, 2011) attempted to enlarge the former analysis by dropping some of these assumptions. By restricting our analysis to tax revenue maximizer and establishing the supermodularity of the tax competition game, we circumvent some difficulties stressed in previous articles to establish the existence of a Nash equilibrium.

Following the contraction approach[17] we establish a sufficient condition for the uniqueness of the Nash equilibrium when the game is supermodular.

Proposition 2. *Under Assumptions 1 to 4, the Nash equilibrium of the tax competition game*

[16]Since Proposition 1 means the supermodularity of $R_i(t)$, it involves also the strategic substitutability of tax rates for the fixed factor income. Let denote by $\Psi_i(t)$ the fixed factor income: $\Psi_i(t) \equiv f_i(k_i) - k_i f_i'(k_i)$. The maximization of $W_i(t)$ with respect to t_i induces $k_i f_i''(k_i) \frac{\partial k_i}{\partial t_i} = -\frac{\partial \Psi_i(t)}{\partial t_i} = \frac{\partial R_i(t)}{\partial t_i}$ and consequently $\frac{\partial^2 R_i(t)}{\partial t_i \partial t_j} > 0 \Leftrightarrow \frac{\partial^2 \Psi_i(t)}{\partial t_i \partial t_j} < 0$.

[17]See Vives (1999, p. 46-48) for an application of this approach in industrial organization.

(Γ) is unique if

$$\forall i \in \{1,...,N\}\,,\ \forall k_i \in \left[0,\bar{k}\right],\ \frac{1}{f_i''(k_i)} < \sum_{j=1,j\neq i}^{n} \frac{1}{f_j''(k_j)}. \tag{12}$$

Proof. See Appendix A.4. □

The sufficient condition (12) means that the marginal capital demand in one country cannot exceed the total marginal demand for capital of all the countries. This condition is also equivalent to establish a lower bound in absolute value on the impact of each country's tax policy on the net return of capital: $\frac{\partial r}{\partial t_i} > -\frac{1}{2}$. In the case of symmetric countries inequality (12) always holds.

4 Some consequences of the supermodularity of the tax competition game

We present in this section some interesting results contributing to the theoretical debate about tax competition and coordination and deriving from the supermodularity of the tax competition game.[18] These results hold for different payoff functions, especially welfare functions, and for other types of tax competition (e.g., commodity tax competition), as long as these games are supermodular.

First, we distinguish two notions often confused in the literature: tax coordination and tax cooperation. The proposed distinction will clarify some consequences of the supermodularity of the tax competition game. Following the literature on macroeconomic games (Cooper, 1999), we say that there is a tax coordination problem when countries fail to reach the Pareto dominant Nash equilibrium of the tax competition game (Γ). This definition suggests that a tax coordination problem emerges when the two following conditions are met: (*i*) Nash equilibriums are multiple and (*ii*) they can be Pareto ranked.[19] Tax cooperation consists of reaching a Pareto superior outcome, which does not need to be a Nash equilibrium of the game (Γ).[20] For instance, Keen and Wildasin (2004) consider tax cooperation by applying the Motzkin's theorem to determine under which conditions a Pareto improving tax reform exists. Given our previous definitions, tax harmonization seems to be more a tax cooperation issue than a tax coordination one: identical tax rates may occur at a Pareto dominant Nash equilibrium, but such a case will be very fortuitous; it seems more realistic to consider Pareto improving tax harmonization, when it exists,

[18] Most of these results have already been established and used in game theory and in industrial organization.

[19] Such a definition restricts the scope of coordination failure. Indeed, the stag hunt game is an example of coordination failure, while the battle of sex game is not since the second criterion is not respected.

[20] The well-known prisonner dilemma illustrates a cooperation failure.

as an outcome, which is not a Nash equilibrium of the initial tax competition game (Γ). The following Corollary derives from our definition of tax coordination:

Corollary 3. *If the tax competition game (Γ) is supermodular, tax coordination is Pareto improving.*

Proof. From Milgrom and Roberts (1990) or Vives (1990), we know that the Nash equilibriums of the tax competition game (Γ) are Pareto ordered with a minimal and a maximal equilibrium since tax revenue is increasing in the tax rates of other countries (plain complementarity property). □

Tax coordination consists of switching from one Nash equilibrium to a Pareto dominant one and it is unambiguously Pareto improving by definition. It assumes obviously that countries are currently locked on the *bad* Nash equilibrium (low tax rate and low tax revenue), which would correspond to the harmful tax competition in OECD's terminology (OECD, 1998).

Several ways to coordinate or to cooperate have been explored in game theory and industrial organization. A first simple instrument available to countries is communicating on their respective tax policy. The game (Γ) is then extended by allowing some cheap talk or costless communication before any tax policy decision. Cheap talk would solve the tax coordination failure if and only if the tax competition game (Γ) displays two necessary credibility properties: the self-committing condition as defined by Farrell (1988) and the self-signaling condition, a stronger requirement emphasized by Aumann (1990). We establish the following Corollary:[21]

Corollary 4. *If the tax competition game (Γ) is supermodular, cheap talk does not enable tax coordination nor tax cooperation.*

Proof. See Appendix A.5. □

In presence of strategic and plain complements each country has an incentive to induce other countries to raise their respective tax rate and trigger a profitable capital inflow, which increases its tax revenue.

The pre-play stage may be also the opportunity for some commitments. For instance, in a two-country tax competition game Kempf and Rota-Graziosi (2010) consider that countries are able to commit themselves to fix their respective tax rate earlier or later. This allows the authors to apply to tax competition the endogenous timing game as proposed by Hamilton and

[21] Baliga and Morris (2002) established this result for games with strategic complementairties and positive spillovers.

Slutsky (1990). Assuming the supermodularity of the tax competition game, Kempf and Rota-Graziosi (2010) establish a ranking of tax rates at the Nash equilibrium of the static game and at the two Stackelberg equilibriums. Plain and strategic complementarities involve that both countries have a second-mover advantage and a first-mover incentive.[22] They deduce that the two Stackelberg outcomes are the perfect subgame Nash equilibrium of the endogenous timing game. In other words, the simultaneous Nash equilibrium of the tax competition game is not anymore commitment robust, when the game is supermodular. Rota-Graziosi (2015) extends this approach by studying a broader commitment device, which consists of voluntary restrictions of countries' strategy sets: at the pre-play stage, each country is able to rule out some actions, i.e. some values of its respective tax rates before tax competition takes place. Tax coordination and even tax harmonization are Nash implementable through this form of commitments, which is self-enforcing.

Beyond communication or commitment about their respective tax policy, some countries may form coalition to avoid harmful tax competition. An example is the Enhanced Cooperation Agreements for European member states proposed in the treaties of Amsterdam (1997) and Nice (2003). Such schemes would correspond to partial tax cooperation. The cooperation among a subset of countries aims at reaching a Pareto superior situation, which is not a Nash equilibrium of the initial tax competition game (Γ). Applying one of the results of Milgrom and Roberts (1996), we obtain:

Corollary 5. *If the tax competition game (Γ) is supermodular, the coalition of a subset of countries does not enable tax cooperation.*

Proof. Milgrom and Roberts (1996, Theorem A.2, page 127) establish that the highest Nash equilibrium of a game with plain and strategic complements is (strongly) coalition proof. □

Partial tax cooperation, that is a situation where a subgoup of countries cooperate cannot be a Nash equilibrium if the tax competition game (Γ) is supermodular. This result completes previous analyses (Keen and Konrad, 2013) and contrasts with Konrad and Schjelderup (1999). Indeed, Milgrom and Roberts (1996) consider explicitly internal and external stability properties of coalitions.

Finally, an interesting consequence of the supermodularity of the tax competition game consists

[22] A player has a first-mover incentive when she prefers to be the leader in the corresponding Stackelberg game than to play the static game. She has a second-mover advantage when she prefers to follow than to lead.

of appreciating the effect of the number of competing jurisdictions on Nash equilibrium tax rates for a given stock of capital. This issue refers to the economic analysis of municipality mergers and more broadly to optimal decentralization. We obtain the following proposition:

Proposition 3. *If the tax competition game (Γ) is supermodular, any increase in the number of active tax-competing countries (N) for a given stock of capital ($\bar{k}$) reduces Nash equilibrium(s) tax rates (t_i) and tax revenues (R_i), and increases the equilibrium net return of capital (r).*

Proof. See Appendix A.6. □

Proposition 3 provides an alternative view to Bucovetsky (2009), who analyzes tax competition intensity.[23] It also completes the analysis of Hoyt (1991), who shows that a decrease in the number of identical competing jurisdictions increases the equilibrium tax rates by improving their respective market power. Our result suggests also an indirect empirical test of the strategic complementarity of tax rates. Indeed, if tax rates are strategic substitutes, then any increase in the number of countries or jurisdictions would imply an increase in tax rates (see Corchon, 1994 for game with strategic substitutes). The increase in the number of tax competing countries may also be approximated by a higher mobility of capital.

5 Limits to the supermodularity of the tax competition game: The role of capital supply

One obvious limit of our analysis is the choice of the payoff function: tax revenue. Establishing similar conditions for the supermodularity of the tax competition game with welfare functions would imply several additional restrictive assumptions. For instance, the choice of the type of public consumption as substitute (public good) or complement (public input) of private income would affect the nature of interactions between countries. The degree of the marginal rate of substitution between private and public consumption would also modify the properties of the tax competition game, and tax rates may become strategic substitutes as highlighted by de Mooij and Vrijburg (2016) with quadratic production functions. However, as noted in the introduction, considering public goods yields implicitly to study a fiscal competition framework, where public income and spending are taken into account, rather than a strict tax competition model.

[23] Bucovetsky (2009) considers welfare maximizers with quadratic production functions, Edgeworth independence between private and public consumption, and fixed capital supply. With this formalization any merger of countries implies an increase in the average tax rate.

Besides the type of public good, we focus on the role of capital supply, which would have to be taken into account with welfare maximizers. First, we address the issue of capital supply by relaxing the implicit assumption of its inelasticity at the level of the economy. We assume a positive relationship between total capital supply ($\overline{k}$) and its net return (r) as in Eichner and Runkel (2012). A common pool problem emerges: any tax rate's increase in one country reduces the net return of capital and consequently total capital supply. Saving decisions and the choice of tax rate occur simultaneously, allowing us avoiding the time inconsistency problem as pointed out by Kydland and Prescott (1977). We have:[24]

$$\overline{k} \equiv S(r)\,, \text{ with } \quad S'(r) > 0 > S''(r)\,.$$

The market-clearing conditions given in (5) are modified consequently:

$$\begin{cases} f_i'(k_i) - t_i = r, \quad \forall i \in \{1, ..., n\} \\ \sum\limits_{i=1}^{n} k_i = S(r) \end{cases} \tag{13}$$

where $\overline{k}(t) = \overline{k}(t_1, ..., t_n)$ is decreasing in tax rate: $\forall i \in \{1, ..., n\}\,, \frac{\partial \overline{k}(t)}{\partial t_i} < 0$. This yields an additional effect: the global tax base contracts in reaction to any increase of tax rate. Expression (7) becomes

$$\frac{\partial r}{\partial t_i} = -\frac{\frac{1}{f_i''(k_i)}}{\sum\limits_{l=1}^{n} \frac{1}{f_l''(k_l)} - S'(r)} < 0, \quad \text{since } S'(r) > 0. \tag{14}$$

Thus, we obtain:[25]

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} \gtrless \Omega - \left(S'(r)\,\mathbf{r} + S''(r)\right) \frac{f_i''(k_i)}{f_j''(k_j)} t_i \left(\frac{\partial r}{\partial t_i}\right)^3 \gtrless 0. \tag{15}$$

The supermodularity of the tax competition game with an endogenous stock of capital imposes an additional constraint on the shape of the saving function. The convexity of the latter must be bounded to preserve the supermodularity of the tax competition game. We remark that not only is the strategic substitutability of tax rates possible for some countries maximizing their tax revenue, but also that some may have non-monotone best replies even under Assumptions

[24]Saving is a function of the interest rate and is increasing with the rate of net return ($S'(r) > 0$) as a result of the concavity of the utility function: $S'(r) = -1/U''$. Moreover, the saving function is concave owing to the non negative absolute risk aversion captured by $S''(r) = -U'''/(U'')^2$.

[25]$\Omega \equiv \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \left(1\text{-}\mathbf{r}t_i - \mathbf{r}t_i \frac{\partial r}{\partial t_i}\right) - \frac{1}{f_j''(k_j) f_i''(k_i)} t_i \left(\frac{\partial r}{\partial t_i}\right)^2$. The proof is available upon request.

1 to 4.

A second issue, we consider here, is capital ownership, which induces a pecuniary effect in an opposite way from the tax base effect. Indeed, any increase in the tax rate of one country reduces the worldwide net return of capital and hurts all capital owners. The payment function for capital owners is denoted by $H_i(t)$: $H_i(t) \equiv r(t)\,\theta_i\bar{k}$, where θ_i is the share of capital owned by inhabitants of country i. Assuming an inelastic capital supply ($\bar{k}$) for simplicity purpose, we have

$$\frac{\partial H_i(t)}{\partial t_j} = \theta_i\bar{k}\frac{\partial r(t)}{\partial t_j} < 0 \text{ and } \frac{\partial^2 H_i(t)}{\partial t_i \partial t_j} = \theta_i\bar{k}\frac{\partial^2 r(t)}{\partial t_i \partial t_j} \lesseqgtr 0.$$

Introducing capital ownership in the objective function may not only cancel the properties of plain and strategic complementarities but also the monotonicity of the payoff function with respect to the action of the other countries and the monotonicity of best replies. Moreover adding capital ownership implies to study its distribution within and between countries. The tax competition game becomes more complex, since it addresses not only an efficiency issue, but also an equity one by stressing the redistibutive implications of tax competition.

Offshore financial centers or tax havens characterized by a zero capital tax rate and no real economic activity,[26] may be represented by the objective function $H_i(t)$. They are specific players: they display plain substitute and may be characterized by strategic substitutability. Consequently, they modify drastically the nature of international tax competition as emphasized by Slemrod and Wilson (2009), Johannesen (2010), Keen and Konrad (2013), and Bucovetsky (2014). Tax havens provide an opportunity to capital owners to protect their interests by improving the net return of capital through a more intensive tax competition. Indeed let assume that a country i initially represented by the payoff function $R_i(t)$ becomes a tax haven characterized by the function $H_i(t)$, the optimal tax rate of this country, which was initially strictly positive becomes zero.[27] Given the strategic complementarity of the tax rates for all the other $N-1$ countries, this variation means a decrease in their respective equilibrium tax rates and then an increase in the net return of capital (r). If we assume an elastic capital supply with respect to r, the emergence of a tax haven induces a higher level of capital and may even increase tax revenues for some non-tax-haven countries (see Desai, Foley, and Hines, 2004 and Dharmapala, 2008 who highlight the positive impact of offshore centers on neighboring economies). It

[26] We do not consider here secrecy, which is another characteristic of many tax havens and tax evasion.

[27] From the First Order Condition of the maximization of $R_i(t)$ with respect to t_i, we deduce that t_i is strictly positive as long as k_i is not equal to zero. Considering $k_i = 0$ is equivalent to saying that country i does not participate to the tax competition game. We excluded the case of negative tax rates.

is worthwile to note that despite the decrease in statutory Corporate Income Tax (CIT) rates accross the world and the increase in Foreign Direct Investment (FDI) flows to tax haven a sharp decline of CIT revenues has not been observed (see IMF, 2014). Finally, we remark that if the supermodularity of the tax competition game does not hold anymore due to the presence of tax havens, then the "race to the bottom" may not exist, tax coordination as previously defined would become impossible, and coalition of a subgroup of countries (tax haven or not) may be Pareto improving. A formalization of previous relationships imposes a general analysis of games with strategic complements and substitutes, which remains for future research.

6 Conclusion

Is tax competition harmful? Can coordination be Pareto improving? We address these questions by establishing sufficient conditions for the supermodularity of the tax competition game in which countries maximize their tax revenue. Even with this simple setup, we emphasize that any increase in the tax rate of one country has two opposite effects on the reaction of the other countries: (*i*) it decreases the woldwide net return of capital allowing the other countries to raise their own tax rates *ceteris paribus*, which induces the strategic complementarity of tax rates; (*ii*) but, this increase involves a reallocation of the capital: the tax base effect (positive tax spillovers), since capital outflows the country, which raises its tax rate, and reduces the marginal capital productivity of the other countries (given the concavity of production functions) and brings them to reduce their tax rate (strategic substitutability).

The $\mathbf{r}-$concavity of marginal productivity function with $\mathbf{r} \leqslant 1$ is a sufficient condition for the strategic complementarity of tax rates and then the supermodularity of the tax competition game. This condition bounds the shape of countries' demand for capital. The quadratic production function respects all our assumptions. Regarding Cobb-Douglas function we have an additional condition binding the stock of capital to the parameters of the function. The existence of at least one Nash equilibrium follows from the supermodularity of the tax competition game. The Nash equilibrium is unique if the marginal effect of tax rate on the net return of capital remains moderate. In case of multiple Nash equilibriums, they can be Pareto ranked given the existence of positive tax spillovers. The "race to the bottom" corresponds to a coordination failure on a bad equilibrium: low tax rate and low tax revenue. If the tax competition game is supermodular, tax coordination is unambiguously Pareto improving; cheap talk is ineffective to allow this coordination, while some commitment devices may be a solution; tax cooperation is not possible through the coalition of some subsets of countries since the highest Nash equilibrium is coalition-proof; finally, any increase in the number of tax-competing countries for a given stock of capital reduces equilibrium tax rates and tax revenue and improves the net return of capital.

Considering welfare maximizers raises multiple issues. Beyond the complementarity or substitutability of public and private consumptions, we highlight the effect of endogenous capital supply and capital ownership on the nature of tax competition. Supermodularity is still possible when the total stock of capital depends on its net return. However, this property may vanish completely when capital ownership is concentrated in some particular jurisdictions such as offshore financial centers or tax havens. Despite its simplicity: tax revenue maximizer and capital tax rate competition, our formalization displays the potential complexity of any tax competition game. As mentioned in the introduction, this paper is a preliminary stage toward a deeper application of the supermodularity tools to tax system competition, in particular to apprehend the multidimensionality of tax systems (see for instance Bucovetsky, 1991, who studies capital and labor taxation, or Cremer and Gahvari 2000, who consider tax and audit rate as policy variables) or the redistributive impact of tax competition. Finally, a natural extension of this work would be to study sufficient condition for the quasi-supermodularity of the tax competition game following the generalization provided by Milgrom and Shannon (1994).

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Appendix

A.1 Second Order Conditions

The Second Order Condition of (2) is given by

$$\frac{\partial^2 R_i}{\partial t_i^2} = 2\frac{\partial k_i}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i^2} = \frac{2}{f_i''(k_i)}\left(1 + \frac{\partial r}{\partial t_i}\right) + t_i \frac{\partial^2 k_i}{\partial t_i^2}, \tag{16}$$

where

$$\frac{\partial^2 k_i}{\partial t_i^2} = -\frac{f_i'''(k_i)}{(f_i''(k_i))^2}\left(1 + \frac{\partial r}{\partial t_i}\right) + \frac{1}{f_i''(k_i)}\frac{\partial^2 r}{\partial t_i^2}. \tag{17}$$

We consider $\frac{\partial^2 r}{\partial t_i^2}$. We have

$$\frac{\partial^2 r}{\partial t_i^2} = -\frac{-\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial k_i}{\partial t_i}\sum\limits_{l=1}^{n}\frac{1}{f_l''(k_l)} + \frac{1}{f_i''(k_i)}\sum\limits_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2}\frac{\partial k_l}{\partial t_i}}{\left(\sum\limits_{l=1}^{n}\frac{1}{f_l''(k_l)}\right)^2},$$

or equivalently

$$\begin{aligned}\frac{\partial^2 r}{\partial t_i^2} &= f_i'''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^2\frac{\partial k_i}{\partial t_i}\sum_{l=1}^{n}\frac{1}{f_l''(k_l)} - \frac{f_i'''(k_i)}{f_i''(k_i)}\left(\frac{\partial r}{\partial t_i}\right)^2\frac{\partial k_i}{\partial t_i} \\ &\quad - f_i''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^2\sum_{l=1,l\neq i}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2}\frac{\partial k_l}{\partial t_i}.\end{aligned}$$

From (7): $f_i''(k_i)\frac{\partial r}{\partial t_i}\sum\limits_{l=1}^{n}\frac{1}{f_l''(k_l)} = -1$, we obtain

$$\frac{\partial^2 r}{\partial t_i^2} = -\frac{f_i'''(k_i)}{f_i''(k_i)}\frac{\partial r}{\partial t_i}\frac{\partial k_i}{\partial t_i}\left(1 - \frac{\partial r}{\partial t_i}\right) - f_i''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^2\sum_{l=1,l\neq i}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2}\frac{\partial k_l}{\partial t_i}.$$

Using (8) the previous expression is equivalent to

$$\frac{\partial^2 r}{\partial t_i^2} = -\frac{f_i'''(k_i)}{(f_i''(k_i))^2}\frac{\partial r}{\partial t_i}\left[1 - \left(\frac{\partial r}{\partial t_i}\right)^2\right] - f_i''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^3\sum_{l=1,l\neq i}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3} \geqslant 0,$$

which induces $\partial^2 k_i/\partial t_i^2 \leqslant 0$ and the respect of the SOCs.

A.2 Proof for Proposition 1: The strategic complementarity of tax rates

The game $\Gamma \equiv (T_i, R_i; i \in \mathbf{N})$ is supermodular if (1) T_i is a compact set in $\mathbb{R}$ ($t_i \in [0,1]$); (2) $R_i(.)$ displays strategic complementarity in tax rates since the strategy set is one-dimensional. The first condition always holds since we consider $T_i = [0,1]$. The second condition yields to sign the cross derivative of tax revenue, which is given by

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{\partial k_i}{\partial t_j} + t_i \frac{\partial^2 k_i}{\partial t_i \partial t_j}.$$

From (8), we deduce that

$$\begin{aligned}
\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial k_i}{\partial t_j}\left(1+\frac{\partial r}{\partial t_i}\right) + \frac{1}{f_i''(k_i)}\frac{\partial^2 r}{\partial t_i \partial t_j} \\
&= -\frac{1}{f_j''(k_j)}\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial r}{\partial t_i}\left(1+\frac{\partial r}{\partial t_i}\right) + \frac{1}{f_i''(k_i)}\frac{\partial^2 r}{\partial t_i \partial t_j}.
\end{aligned}$$

Given (7), we obtain

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = \frac{\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial k_i}{\partial t_j}\sum_{l=1}^{n}\frac{1}{f_l''(k_l)} - \frac{1}{f_i''(k_i)}\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2}\frac{\partial k_l}{\partial t_j}}{\left(\sum_{l=1}^{n}\frac{1}{f_l''(k_l)}\right)^2},$$

Given the definition of $\frac{\partial r}{\partial t_i}$ in (7), the previous expression is also equal to

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = -\frac{f_i'''(k_i)}{f_i''(k_i)}\frac{\partial r}{\partial t_i}\frac{\partial k_i}{\partial t_j} - f_i''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^2 \sum_{l=1}^{n}\left[\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2}\frac{\partial k_l}{\partial t_j}\right],$$

which corresponds to[28]

$$\begin{aligned}
\frac{\partial^2 r}{\partial t_i \partial t_j} &= -\frac{f_i'''(k_i)}{f_i''(k_i) f_j''(k_i)}\left(\frac{\partial r}{\partial t_i}\right)^2 - \frac{\left(f_i''(k_i)\right)^2}{f_j''(k_j)}\left(\frac{\partial r}{\partial t_i}\right)^3 \sum_{l=1, l\neq j}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3} \\
&\quad - f_i''(k_i)\frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3}\left(1+\frac{f_i''(k_i)}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\right)\left(\frac{\partial r}{\partial t_i}\right)^2
\end{aligned}$$

[28] $\frac{\partial k_i}{\partial t_j} = \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i}$, $\frac{\partial k_{l\neq j}}{\partial t_j} = \frac{1}{f_l''(k_l)}\frac{f_i''(k_i)}{f_j''(k_j)}\frac{\partial r}{\partial t_i}$, and $\frac{\partial k_j}{\partial t_j} = \frac{1}{f_j''(k_j)}\left(1+\frac{\partial r}{\partial t_j}\right) = \frac{1}{f_j''(k_j)}\left(1+\frac{f_i''(k_i)}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\right)$.

or equivalently,

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = -\frac{f_i''(k_i)}{f_j''(k_j)}\left(\frac{\partial r}{\partial t_i}\right)^2\left(\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} + f_i''(k_i)\frac{\partial r}{\partial t_i}\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\right). \tag{18}$$

We deduce that

$$\begin{aligned}\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\frac{f_i'''(k_i)}{f_j''(k_j)(f_i''(k_i))^2}\frac{\partial r}{\partial t_i}\left(1+\frac{\partial r}{\partial t_i}\right)\\ &-\frac{1}{f_j''(k_j)}\left(\frac{\partial r}{\partial t_i}\right)^2\left(\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} + f_i''(k_i)\frac{\partial r}{\partial t_i}\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\right). \end{aligned} \tag{19}$$

We have then

$$\begin{aligned}\frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\left(1-\frac{f_i'''(k_i)}{(f_i''(k_i))^2}t_i\left(1+\frac{\partial r}{\partial t_i}\right)\right)\\ &-\frac{1}{f_j''(k_j)}t_i\left(\frac{\partial r}{\partial t_i}\right)^2\left(\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} + f_i''(k_i)\frac{\partial r}{\partial t_i}\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\right),\end{aligned}$$

or equivalently

$$\begin{aligned}\frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\left(1-\frac{f_i'''(k_i)}{(f_i''(k_i))^2}t_i\right)\\ &-\frac{1}{f_j''(k_j)}t_i\left(\frac{\partial r}{\partial t_i}\right)^2\left(2\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} + f_i''(k_i)\frac{\partial r}{\partial t_i}\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\right)\end{aligned} \tag{20}$$

From the assumption of the $\mathbf{r}-$concavity of $f_i'(.)$ we deduce that

$$-\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2} \geqslant -\mathbf{r} \textit{ and } \frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^3} \geqslant \frac{\mathbf{r}}{f_i''(k_i)}.$$

Expression (20) becomes then

$$\begin{aligned}\frac{\partial^2 R_i}{\partial t_i \partial t_j} &\geqslant \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\left(1-\mathbf{r}t_i\left(1+\frac{\partial r}{\partial t_i}\right)\right)\\ &-\frac{1}{f_j''(k_j)}t_i\left(\frac{\partial r}{\partial t_i}\right)^2\left(2\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} + \mathbf{r}f_i''(k_i)\frac{\partial r}{\partial t_i}\sum_{l=1}^{n}\frac{1}{f_i''(k_i)}\right).\end{aligned}$$

Using (7) we obtain

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geqslant \frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i}\left(1-\mathbf{r}t_i\left(1+2\frac{\partial r}{\partial t_i}\right)\right)-\frac{1}{f_j''(k_j)}t_i\left(\frac{\partial r}{\partial t_i}\right)^2\left(\frac{f_i'''(k_i)}{(f_i''(k_i))^2}+\frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2}\right).$$

We deduce the sufficient condition for $\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geqslant 0$, which is $\mathbf{r} \leqslant 1$. Indeed, given that $t_i \in [0,1]$,[29] we have

$$1-\mathbf{r}t_i\left(1+2\frac{\partial r}{\partial t_i}\right) \geqslant 1-\mathbf{r}t_i-2\mathbf{r}t_i\frac{\partial r}{\partial t_i} \geqslant 1-\mathbf{r} \geqslant 0 \; if \; \mathbf{r} \leqslant 1.$$

A.3 Proof of Corollary 1: The strategic complementarity of tax rates with Cobb-Douglas production function

The Cobb-Douglas function respects Assumptions (1) and (2): $f_i(k_i) = A_i k_i^{\alpha_i}$, with $A_i > 0$ and $0 < \alpha_i < 1$, $f_i'(k_i) = A_i\alpha_i k_i^{\alpha_i-1} \geqslant 0$, $f_i''(k_i) = A_i\alpha_i(\alpha_i-1)k_i^{\alpha_i-2} < 0$, $f_i'''(k_i) = A_i\alpha_i(\alpha_i-1)(\alpha_i-2)k_i^{\alpha_i-3} \geqslant 0$. Applying Lemma 1, we obtain an additional condition for the $\mathbf{r}-$concavity of Cobb-Douglas function: Let define $\underline{A} \equiv \underset{i\in N}{min}\{A_i\}$, $\underline{\alpha} \equiv \underset{i\in N}{min}\{\alpha_i\}$, and $\bar{\alpha} \equiv \underset{i\in N}{max}\{\alpha_i\}$, we have:
$\forall i \in \{1,...,N\}$, $\forall k_i \in \left[0,\overline{k}\right]$,

$$\frac{1}{A_i}\frac{2-\alpha_i}{\alpha_i(1-\alpha_i)}k_i^{1-\alpha_i} \leqslant \frac{1}{\underline{A}}\frac{2-\underline{\alpha}}{\underline{\alpha}(1-\bar{\alpha})}\bar{k}^{1-\underline{\alpha}} \equiv \mathbf{r}^* < \infty,$$

and

$$\mathbf{r}^* \leqslant 1 \Leftrightarrow \bar{k}^{1-\underline{\alpha}} \leqslant \underline{A}\frac{\underline{\alpha}(1-\bar{\alpha})}{2-\underline{\alpha}}.$$

We note that the previous condition involves Assumption 3 since

$$\forall i \in \{1,...,N\}, \quad f_i'\left(\bar{k}\right) = A_i\alpha_i\bar{k}^{\alpha_i-1} \geqslant \underline{A\alpha}\bar{k}^{\bar{\alpha}-1} > 1,$$

which is equivalent to

$$\bar{k}^{1-\bar{\alpha}} < \underline{\alpha A}.$$

Finally, we note that $\bar{k}^{1-\bar{\alpha}} \leqslant \bar{k}^{1-\underline{\alpha}} \leqslant \underline{\alpha A}\frac{1-\bar{\alpha}}{2-\underline{\alpha}} < \underline{\alpha A}$. Condition (11) involves then Assumption 3.

[29]The tax rate cannot be negative at the equilibrium. Indeed, we have: $\left.\frac{\partial R_i}{\partial t_i}\right|_{t_i=0} = k_i \geqslant 0$.

A.4 Proof for Proposition 2: Uniqueness of the Nash equilibrium

We follow the contraction approach to establish the uniqueness of the Nash equilibrium. By application of Proposition 1, the game Γ is supermodular and the uniqueness of the Nash equilibrium may be deduced from:

$$\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1,j\neq i}^{n} \left|\frac{\partial^2 R_i}{\partial t_i \partial t_j}\right| = \frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1,j\neq i}^{n} \frac{\partial^2 R_i}{\partial t_i \partial t_j} < 0.$$

We have

$$\begin{aligned}
\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1,j\neq i}^{n} \frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{2}{f_i''(k_i)}\left(1+\frac{\partial r}{\partial t_i}\right) + t_i \frac{\partial^2 k_i}{\partial t_i^2} + \sum_{j=1,j\neq i}^{n}\left(\frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i \partial t_j}\right) \\
&= \frac{1}{f_i''(k_i)}\left(2+\frac{\partial r}{\partial t_i}\right) + \sum_{j=1}^{n}\left(\frac{1}{f_j''(k_j)}\frac{\partial r}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i \partial t_j}\right).
\end{aligned}$$

Applying (7), we obtain

$$\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1,j\neq i}^{n} \frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{1}{f_i''(k_i)}\left(1+\frac{\partial r}{\partial t_i}\right) + t_i \sum_{j=1}^{n} \frac{\partial^2 k_i}{\partial t_i \partial t_j}.$$

From (19) we have

$$\begin{aligned}
\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3}\left(\frac{\partial r}{\partial t_i}\right)^2 - \frac{1}{f_j''(k_j)}\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial r}{\partial t_i}\left(1+2\frac{\partial r}{\partial t_i}\right) \\
&\quad -\frac{f_i''(k_i)}{f_j''(k_j)}\left(\frac{\partial r}{\partial t_i}\right)^3 \sum_{l=1}^{n} \frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}.
\end{aligned}$$

Adding the latter expression over $j \in N$ we obtain

$$\begin{aligned}
\sum_{j=1}^{n} \frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\left(\frac{\partial r}{\partial t_i}\right)^2 \sum_{j=1}^{n} \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} - \frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^2}\frac{\partial r}{\partial t_i}\left(1+2\frac{\partial r}{\partial t_i}\right)\sum_{j=1}^{n}\frac{1}{f_j''(k_j)} \\
&\quad -f_i''(k_i)\left(\frac{\partial r}{\partial t_i}\right)^3 \sum_{l=1}^{n} \frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\sum_{j=1}^{n}\frac{1}{f_j''(k_j)},
\end{aligned}$$

which can be simplified to

$$\begin{aligned}\sum_{j=1}^{n}\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\left(\frac{\partial r}{\partial t_i}\right)^2\sum_{j=1}^{n}\frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3}+\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^3}\left(1+2\frac{\partial r}{\partial t_i}\right)+\left(\frac{\partial r}{\partial t_i}\right)^2\sum_{l=1}^{n}\frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3}\\ &= \frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^3}\left(1+2\frac{\partial r}{\partial t_i}\right).\end{aligned}$$

We deduce that

$$\frac{\partial^2 R_i}{\partial t_i^2}+\sum_{j=1,j\neq i}^{n}\frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{1}{f_i''(k_i)}\left(1+\frac{\partial r}{\partial t_i}\right)+t_i\frac{f_i'''(k_i)}{\left(f_i''(k_i)\right)^3}\left(1+2\frac{\partial r}{\partial t_i}\right).$$

Since $t_i \geqslant 0$, a sufficient condition is:

$$\forall i\in N,\ \frac{1}{f_i''(k_i)}<\sum_{j=1,j\neq i}^{n}\frac{1}{f_j''(k_j)},$$

which involves

$$\forall i\in N,\ \frac{\partial r}{\partial t_i}>-\frac{1}{2},$$

and then $\frac{\partial^2 R_i}{\partial t_i^2}+\sum_{j=1,j\neq i}^{n}\frac{\partial^2 R_i}{\partial t_i \partial t_j}<0$.

A.5 Proof of Corollary 4

We establish that cheap-talk is neither self-committing nor self-signaling. We have the following definitions:

Definition 1: Action t_i is self-committing if

$$R_i\left(t_i,\tau_j\left(t_i\right)\right)\geqslant R_i\left(t_i',\tau_j\left(t_i'\right)\right)\ \text{ for } \forall t_i'\in[0,1].$$

By definition, only the leader's tax rate at the Stackelberg equilibrium is self-committing on $[0,1]$.

Definition 2: Action t_i is self-signaling if

$$R_i\left(t_i,\tau_j\left(t_i\right)\right)\geqslant R_i\left(t_i',t_j\right)\ \text{ for } \forall\left(t_i',t_j\right)\in[0,1]^2.$$

Given the plain complementarity of the game (Γ), we have:

$$R_i\left(t_i, \tau_j\left(t_i\right)\right) < R_i\left(t_i, t_i'\right) \text{ for } t_i' > \tau_j\left(t_i\right),$$

which contradicts the self-signaling condition.

A.6 Proof for Proposition 3.

First, we sign $\partial t_i/\partial \bar{k}$. Given the market clearing condition (5), we have

$$\sum_{i=1}^{n} f_i'^{-1}\left(r + t_i\right) = \bar{k}.$$

Considering that $\bar{k}$ may vary, we differentiate the previous expression with respect to t_i and obtain:

$$\frac{1}{f_i''\left(k_i\right)}\left(1 + \frac{\partial r}{\partial t_i}\right) = \frac{\partial \bar{k}}{\partial t_i}.$$

Since $f_i''(.) < 0$ and $1 + \partial r/\partial t_i > 0$, we deduce that $\partial \bar{k}/\partial t_i < 0$ or equivalently,

$$\partial t_i/\partial \bar{k} < 0. \tag{21}$$

Secondly, we consider the game with $N+1$ jurisdictions: $\Gamma' \equiv (S_i, R_i; i \in \{1, ..., N+1\})$. We denote respectively the largest Nash equilibrium tax rates[30] of the game Γ by $(t_i(N), t_{-i}(N))$, and this of Γ' by $(t_i(N+1), t_{-i}(N+1))$, where $t_{-i}(N)$ (respectively $t_{-i}(N+1)$) is the vector of equilibrium tax rates for the $N-1$ (resp. N) other countries j with $(j \neq i)$. Given the market clearing condition (5) and under the assumption that the additional jurisdiction $(N+1)$ is active, this jurisdiction attracts some capital $k_{N+1}(t_{N+1}(N+1)) > 0$. Consequently, the N remaining jurisdictions compete for a lower stock of capital than without the jurisdiction $N+1$: $\tilde{k} \equiv \bar{k} - k_{N+1}(.) < \bar{k}$. From (21) we deduce that any increase in the number of active jurisdictions reduces the equilibrium tax rates.

Focusing on tax revenue we have: $\forall i \in \{1, ..., N\} \cap \{1, ..., N+1\}$,

$$\begin{aligned} R_i\left(t_i(N), t_{-i}(N)\right) &= \max_{t_i \in [0,1]} R_i\left(t_i, t_{-i}(N)\right) \geqslant R_i\left(t_i(N+1), t_{-i}(N)\right) \\ &\geqslant R_i\left(t_i(N+1), t_{-i}(N+1)\right). \end{aligned}$$

[30]The proof for the smallest Nash equilibrium is similar.

where the first inequality results from the definition of the Nash equilibrium and the second from the fact that $t_{-i}(N) \geqslant t_{-i}(N+1)$ and the plain complementarity of tax rates $(\partial R_i(.)/\partial t_j \geqslant 0)$. Finally, the negative relationship between r and N can be derived from $\partial \bar{k}/\partial t_i$ and $\partial r/\partial t_i$.

The strategic complementarity of tax rates with an endogenous stock of capital (not for publication)

This Appendix concerns expression (15) in the main text. Differentiating expression (14) with respect to t_j yields

$$
\begin{aligned}
\frac{\partial^2 r}{\partial t_i \partial t_j} &= -\frac{f_i'''(k_i)}{f_i''(k_i)} \frac{1}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \\
&\quad - f_i''(k_i) \left(\sum_{l=1, l\neq j}^{n} \frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^2} \frac{1}{f_l''(k_l)} \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right. \\
&\quad \left. + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^2} \frac{1}{f_j''(k_j)} \left(1 + \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i}\right) + S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) \left(\frac{\partial r}{\partial t_i}\right)^2 \\
&= -\frac{f_i'''(k_i)}{f_i''(k_i)} \frac{1}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \\
&\quad - f_i''(k_i) \left(\frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \sum_{l=1}^{n} \frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} + S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) \left(\frac{\partial r}{\partial t_i}\right)^2
\end{aligned}
$$

Substituting this expression in $\frac{\partial^2 k_i}{\partial t_i \partial t_j}$ we have

$$
\begin{aligned}
\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) - \frac{1}{f_i''(k_i) f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \\
&\quad - \left(\frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \sum_{l=1}^{n} \frac{f_l'''(k_l)}{\left(f_l''(k_l)\right)^3} + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} + S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) \left(\frac{\partial r}{\partial t_i}\right)^2 \\
&= -\frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) - \frac{1}{f_i''(k_i) f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2
\end{aligned}
$$

Applying Assumption 4, we obtain

$$\begin{aligned}
\frac{\partial^2 k_i}{\partial t_i \partial t_j} \geqslant\; & -\frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i}\left(1+\frac{\partial r}{\partial t_i}\right) - \frac{1}{f_i''(k_i) f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \\
& -\left(\frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \mathbf{r} \left[\sum_{l=1}^{n} \frac{1}{f_l''(k_l)} - S'(r) \right] + S'(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \mathbf{r} \right. \\
& \left. + \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} + S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) \left(\frac{\partial r}{\partial t_i}\right)^2 \\
=\; & -\frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i}\left(1+\frac{\partial r}{\partial t_i}\right) - \frac{1}{f_i''(k_i) f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \\
& + \left(\frac{1}{f_j''(k_j)} \mathbf{r} - \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} - S'(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \mathbf{r} - S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) \left(\frac{\partial r}{\partial t_i}\right)^2,
\end{aligned}$$

which yields

$$\begin{aligned}
\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geqslant\; & \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} - \frac{1}{f_j''(k_j)} \mathbf{r} t_i \frac{\partial r}{\partial t_i}\left(1+\frac{\partial r}{\partial t_i}\right) - \frac{1}{f_i''(k_i) f_j''(k_j)} t_i \left(\frac{\partial r}{\partial t_i}\right)^2 \\
& + \left(\frac{\mathbf{r}}{f_j''(k_j)} - \frac{f_j'''(k_j)}{\left(f_j''(k_j)\right)^3} - S'(r) \frac{f_i''(k_i)}{f_j''(k_j)} \mathbf{r} \frac{\partial r}{\partial t_i} - S''(r) \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \right) t_i \left(\frac{\partial r}{\partial t_i}\right)^2.
\end{aligned}$$