Long-term care social insurance. How to avoid big losses?*

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Abstract

Long-term care (LTC) needs are expected to rapidly increase in the next decades and at the same time the main provider of LTC, namely the family is stalling. This calls for more involvement of the state that today covers less than 20% of these needs and most often in an inconsistent way.

Besides the need to help the poor dependent, there is a mounting concern in the middle class that a number of dependent people are incurring costs that could force them to sell all their assets. In this paper we study the design of a social insurance that meets this concern. Following Arrow (1963), we suggest a policy that is characterized by complete insurance above a deductible amount.

Keywords: capped spending, Arrow's theorem, long-term care insurance, optimal taxation

JEL Classification: H21, I13, J14

Introduction

Long-term care (LTC) is becoming a major concern for policy makers. Following the rapid aging of our societies, the needs for LTC are expected to grow and yet there is a lot of uncertainty as how to finance those needs; see Norton (2000) and Cremer, Pestieau and Ponthière (2012) for an overview. Family solidarity, which has been the main provider of LTC, is reaching a ceiling, and the market remains rather thin. Not surprisingly, one would expect that the state takes the relay.

The state plays already some role in most countries but this role is still modest and inconsis- tent. In a recent report for the UK, Andrew Dilnot (2011) sketches the features of what can be considered as an ideal social program for LTC. This would be a two-tier program. The first tier would concern those who cannot afford paying for their LTC. It would be a means-test program. The second tier would address

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the fears of most dependents in the middle class that they might incur costs that would force them to sell all their assets and prevent them from bequeathing any of them. This concern is not met by current LTC practices.

In this paper we want to study the design of a social insurance that would cover those with a modest level of assets (for example 300,000 euros) who can face losing up to their entirety to pay for care costs. To do that we explore Dilnot's suggestion that individuals' contribution to their long-term care costs should be capped at a certain amount, after which they will be eligible for full state support. We are thus in the spirit of Arrow's (1963) theorem on insurance deductible. To recall, this theorem states that "if an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum" (Arrow, 1963). Our paper explores whether and how this idea can be applied to LTC social insurance.

We look at a welfare maximizing government which faces a society consisting of people who differ in their earning and face the risk of dependence. Following Arrow, we assume that insurance is not costless; we thus introduce a loading factor that is at the heart of his theorem. We assume that this is true for both private and social insurance but consider the possibility that the government might face lower costs than private insurers. We study the design of a non-linear optimal social LTC insurance and show that this insurance features a deductible as long as there is a loading cost. We then ask ourselves whether we can obtain maximum social welfare by restricting public policy to income taxation and not interfering in the choice of insurance by individuals, which would be in line with Atkinson and Stiglitz (1976). As it will appear, this result of non interference with the insurance choice of individuals will hold only if individuals have the same probability of losses and the same level of losses. As soon as we depart from this assumption, Atkinson-Stiglitz proposition does not apply and we can tax or subsidize private insurance purchases to improve social welfare. In this paper, we consider two types of individuals: skilled and unskilled. They face a probability of becoming dependent and would like to buy some insurance.When the losses incurred by the skilled are higher than that of the unskilled, there is a case for taxing the premium paid by the unskilled. This tax allows for relaxing the self-selection constraint that the skilled are not tempted to mimic the non skilled. We also use the idea that the higher needs of the skilled are somehow whimsical and thus are not taken seriously by the social planner in his design of optimal policy.

It will be seen in the analysis that the interference or not with individual insurance choices will have an important impact on the way optimal deductibles for skilled and non skilled individuals are designed, but an important role will also be played by absolute risk aversion exhibited by individual preferences.

An insurance policy with deductible is not the only possible type of contract. One of the most common practices today is to provide flat payments. Concretely, the insured individuals are entitled to a (periodic) lump-sum payment conditional on their (observable) degree of dependency. This practice has been justified by Kessler (2008) on the basis of alleged huge ex-post moral hazard and by Cremer et al. (2016) on the basis of family solidarity that acts as a last resort payer. Finally, note that in this paper, we adopt a very simple specification of dependency. We do not explicitly account for the time dimension, namely for the fact that the loss incurred by a dependent depends on the yearly cost of dependency times the number of years of dependency. This number is the difference between the age of death and the age at which an irreversible dependency occurs. For an extension of Arrow's theorem to such a temporal framework, see Drèze et al. (2016).

1 The model

We consider a society consisting of two types of individuals: skilled (i.e. those with a high productivity/wage denoted by w_h) and unskilled (i.e. those with a low productivity/wage $w_l < w_h$). Before their retirement, individuals provide labour supply, respectively l_h and l_l, on the labour market and thus earn respectively $y_h = w_h l_h$ and $y_l = w_l l_l$. By working, the individuals experience a disutility of labour $v(l_i)$ $(i = h, l)$, with $v'(l_i) > 0$ and $v''(l_i) > 0$.

When they reach their old age and retire, the individuals face the risk of becoming dependent. With probability π_1, they experience a low severity level of dependence in which case they have LTC needs (expressed in terms of costs incurred) L_{1i} $(i = h, l)$, with probability π_2, they face a heavy dependence with LTC needs $L_{2i} > L_{1i}$ $(i = h, l)$, and with probability $1 - \pi_1 - \pi_2$, they remain healthy. At each severity level, the two types of individuals can have different LTC needs (i.e. $L_{1h} \neq L_{1l}$ and $L_{2h} \neq L_{2l}$) or these needs can be the same (i.e. $L_{1h} = L_{1l}$ and $L_{2h} = L_{2l}$); we will discuss these cases separately.

The individuals can purchase private LTC insurance which charges a premium P_i and reimburses a fraction α_{1i} of the needs in state 1 and α_{2i} in state 2 ($0 \leq \alpha_{1i} \leq 1$ and $0 \leq \alpha_{2i} \leq 1$; $i = h, l$).[1]

For simplicity, we do not model explicitly the individuals' consumption and saving choices made before the retirement; we rather assume that the individuals save a constant share β of their income left after paying the insurance premium and consume the rest. To simplify even more, we focus on the post-retirement stage and abstract from the individuals' utility of consumption before the retirement. We thus normalize β to 1 and consider that the individuals arrive to the post-retirement stage with a wealth equal to $y_i - P_i$.

The expected utility of an individual i $(i = h, l)$ can thus be written as follows:

$$EU_i = \pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1 - \pi_1 - \pi_2)\, u\left(c_i^{I}\right) - v\left(\frac{y_i}{w_i}\right) \tag{1}$$

where
$c_i^{D_1} = y_i - P_i - (1 - \alpha_{1i})L_{1i}$,
$c_i^{D_2} = y_i - P_i - (1 - \alpha_{2i})L_{2i}$

[1] Following Drèze and Schokkaert (2013), we will show that the equilibrium insurance policy is in line with Arrow's theorem of the deductible.

and $c_i^I = y_i - P_i$ are individual wealth levels in the three states of nature[2]
and $P_i = \pi_1(1+\lambda)\alpha_{1i}L_{1i} + \pi_2(1+\lambda)\alpha_{2i}L_{2i}$, with $\lambda > 0$ being the loading cost of private insurance.

2 The *laissez-faire*

In the *laissez-faire*, the problem of an individual i $(i = h, l)$ is to determine his pre-retirement labour supply l_i (or, equivalently, his earnings y_i) and to choose an insurance policy characterized by a premium P_i and insurance rates α_{1i} and α_{2i} $(0 \leq \alpha_{1i} \leq 1$ and $0 \leq \alpha_{2i} \leq 1)$. The Lagrangean of this problem can be written as follows:

$$\mathcal{L} = \pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1 - \pi_1 - \pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right) + \\ + \mu_i\left[P_i - \pi_1(1+\lambda)\alpha_{1i}L_{1i} - \pi_2(1+\lambda)\alpha_{2i}L_{2i}\right]$$

where, as defined before,
$c_i^{D_1} = y_i - P_i - (1-\alpha_{1i})L_{1i}$,
$c_i^{D_2} = y_i - P_i - (1-\alpha_{2i})L_{2i}$,
$c_i^I = y_i - P_i$
and μ_i is the Lagrange multiplier associated with the constraint defining the insurance premium.
The FOCs with respect to the choice variables are the following:

$$\frac{\partial \mathcal{L}}{\partial y_i} = \pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^I) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\pi_1 u'(c_i^{D_1}) - \pi_2 u'(c_i^{D_2}) - (1-\pi_1-\pi_2)\, u'(c_i^I) + \mu_i = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = u'(c_i^{D_1}) - \mu_i(1+\lambda) \leq 0, \qquad \alpha_{1i}\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = u'(c_i^{D_2}) - \mu_i(1+\lambda) \leq 0, \qquad \alpha_{2i}\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = 0 \tag{5}$$

Following Drèze and Schokkaert (2013), we will now show that the equilibrium insurance policy is in line with Arrow's theorem of the deductible. To see this, first note that from (4), we have that either $\alpha_{1i} = 0$ or $u'(c_i^{D_1}) = \mu_i(1+\lambda)$. It can be easily verified that the second equality is equivalent to

$$(1-\alpha_{1i})L_{1i} = y_i - P_i - u'^{-1}(\mu_i(1+\lambda)).$$

[2]Individuals can obviously decide how to allocate their wealth between, e.g., their old age consumption and bequests left to their children. We do not model these choices explicitly but rather focus on individuals' total wealth. As long as bequests are considered as normal goods, wealthier individuals will leave higher bequests. In other words, individuals want to smooth both their consumption and their bequests across the states of nature.

Similarly, from (5), we have either $\alpha_{2i} = 0$ or

$$(1 - \alpha_{2i})L_{2i} = y_i - P_i - u'^{-1}(\mu_i(1+\lambda)).$$

Denoting $y_i - P_i - u'^{-1}(\mu_i(1+\lambda)) \equiv D_i$, we can write

$$\alpha_{1i} = \max\left[0; \frac{L_{1i} - D_i}{L_{1i}}\right]$$

and

$$\alpha_{2i} = \max\left[0; \frac{L_{2i} - D_i}{L_{2i}}\right]$$

Thus, if the needs are lower than D_i, it is optimal for the individual to have zero insurance coverage and to bear all the costs himself, whereas if the needs are higher than D_i, the optimal insurance is such that the individual actually pays the amount D_i and the rest is covered by the insurer. This is thus exactly what is stated by Arrow's theorem of the deductible.

We therefore have that if the needs are higher than the deductible at both severity levels of dependence (i.e. if all the solutions are interior), the marginal utilities in the two dependence states of nature will be equalized. To compare these marginal utilities with the marginal utility in the state of autonomy, combining (3) with (4) and (5), we get

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)} < 1 \tag{6}$$

We can see that as long as $\lambda > 0$, insurance is not full and thus the deductible is always strictly positive.

For the rest of the analysis, we are going to focus on interior solutions and we are now going to rewrite the above problem in an equivalent way which will allow us to better highlight the connection of our model with Atkinson and Stiglitz (1976).[3]

In particular, we now define three commodities z_i^1, z_i^2 and z_i^0 "consumed" in the three states of nature:

$$z_i^1 \equiv c_i^{D_1} + L_{1i},\ z_i^2 \equiv c_i^{D_2} + L_{2i},\ z_i^0 \equiv c_i^I.$$

The individual problem can then be analyzed in terms of these commodities and labour supply l_i (or earnings y_i). The problem of an individual i thus writes:

$$max\left\{U_i = \pi_1 u\left(z_i^1 - L_{1i}\right) + \pi_2 u\left(z_i^2 - L_{2i}\right) + (1 - \pi_1 - \pi_2)u(z_i^0) - v\left(\frac{y_i}{w_i}\right)\right\}$$

subject to

[3] We are grateful to an anonymous referee for suggesting this approach.

$$y_i \geq (1+\lambda)\pi_1 z_i^1 + (1+\lambda)\pi_2 z_i^2 + (1-(1+\lambda)\pi_1 - (1+\lambda)\pi_2) z_i^0$$

$$\equiv q^1 z_i^1 + q^2 z_i^2 + q^0 z_i^0$$

where $q^1 \equiv (1+\lambda)\pi_1$, $q^2 \equiv (1+\lambda)\pi_2$ and $q^0 \equiv 1-(1+\lambda)\pi_1-(1+\lambda)\pi_2$ can be interpreted as the prices of commodities z_i^1, z_i^2 and z_i^0.

Since the resource constraint is binding in equilibrium, we can express

$$z_i^0 = \frac{y_i - q^1 z_i^1 - q^2 z_i^2}{q^0} \tag{7}$$

Taking into account (7), individual i maximizes U_i with respect to z_i^1, z_i^2 and y_i, which gives the following FOCs:

$$\pi_1 u'(c_i^{D_1}) - (1-\pi_1-\pi_2) u'(c_i^I) \frac{q^1}{q^0} = 0 \tag{8}$$

$$\pi_2 u'(c_i^{D_2}) - (1-\pi_1-\pi_2) u'(c_i^I) \frac{q^2}{q^0} = 0 \tag{9}$$

$$\frac{(1-\pi_1-\pi_2) u'(c_i^I)}{q^0} - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{10}$$

From these FOCs we can then obtain the following optimality conditions:

$$MRS_{z_i^0, z_i^1} \equiv \frac{(1-\pi_1-\pi_2) u'(c_i^I)}{\pi_1 u'(c_i^{D_1})} = \frac{q^0}{q^1}, \tag{11}$$

$$MRS_{z_i^0, z_i^2} \equiv \frac{(1-\pi_1-\pi_2) u'(c_i^I)}{\pi_2 u'(c_i^{D_2})} = \frac{q^0}{q^2}, \tag{12}$$

$$MRS_{l_i, z_i^0} \equiv \frac{v'\left(\frac{y_i}{w_i}\right)}{(1-\pi_1-\pi_2) u'(c_i^I)} = \frac{w_i}{q^0} \tag{13}$$

where MRS denotes the marginal rate of substitution between two commodities. Using the definitions of q^1, q^2 and q^0, it can be easily verified that these conditions imply exactly the same tradeoffs as the ones implied by the interior solutions of the initial specification of the problem.

It should, however, be mentioned that for the present problem to be entirely equivalent to the initial specification, we focus on the solutions with which $z_i^1 - z_i^0 > 0$ and $z_i^2 - z_i^0 > 0$. Note that the difference $c_i^I - c_i^{D_1} = c_i^I - c_i^{D_2}$ can be interpreted as the deductible D_i,[4] which, using the definitions of z_i^1, z_i^2 and

[4] It is easy to see this with the initial specification where $c_i^I = y_i - P_i$ and $c_i^{D_1} = c_i^{D_2} = y_i - P_i - D_i$.

z_i^0, implies that $z_i^1 - z_i^0 = L_{1i} - D_i$ and $z_i^2 - z_i^0 = L_{2i} - D_i$. Thus, $z_i^1 - z_i^0 < 0$ and $z_i^2 - z_i^0 < 0$ would imply negative insurance, which is not allowed in the initial specification.

In Appendix A we derive the comparative statics of equilibrium earnings y_i and the three commodities z_i^1, z_i^2 and z_i^0 with respect to changes in the individual's wage/productivity w_i, LTC needs L_{1i} (a change in L_{2i} gives analogous results) and insurance loading cost λ. We then derive the implications of these changes to the size of the deductible faced by the individual i.

We show that y_i always increases with the level of w_i and the same is true for commodities z_i^1, z_i^2 and z_i^0. The levels of z_i^1, z_i^2 and z_i^0 in fact increase with w_i precisely because y_i does. In other words, the increase in the levels of the three commodities is purely triggered by the increase in income.[5] Since the levels of L_{1i} and L_{2i} remain unchanged, the increases of all the three commodities are equivalent to the increases in the levels of consumption in the respective states of nature, i.e. $\frac{\partial z_i^1}{\partial w_i} = \frac{\partial c_i^{D_1}}{\partial w_i}$, $\frac{\partial z_i^2}{\partial w_i} = \frac{\partial c_i^{D_2}}{\partial w_i}$ and $\frac{\partial z_i^0}{\partial w_i} = \frac{\partial c_i^I}{\partial w_i}$. Moreover, it can be easily verified that $\frac{\partial c_i^{D_1}}{\partial w_i} = \frac{\partial c_i^{D_2}}{\partial w_i}$. To see how an increase in w_i affects the deductible faced by the individual i, we need to compare the increases in $c_i^{D_1}$ and $c_i^{D_2}$ to the increase in the healthy state's consumption c_i^I. We show in Appendix A that the difference between these increases depends on the absolute risk aversion (ARA) exhibited by the utility function. More specifically, $c_i^{D_1}$ and $c_i^{D_2}$ increase less than c_i^I under decreasing absolute risk aversion (DARA), more than c_i^I under increasing absolute risk aversion (IARA) and by the same amount as c_i^I under constant absolute risk aversion (CARA) preferences.[6] This implies that the deductible faced by the individual i is increasing (resp. decreasing and constant) in w_i under DARA (resp. IARA and CARA) preferences. This is in line with the deductible insurance theory showing that under DARA (resp. IARA and CARA) the deductible increases (resp. decreases and remains constant) when the initial wealth goes up.[7] Indeed, in our setting, an increase in w_i implies an increase in y_i, which can also be seen as an increase in the initial wealth.

As far as changes in LTC needs are concerned, an increase in L_{1i} fosters labour supply and increases earnings y_i. However, the increase in y_i fostered by a one unit increase in L_{1i} is smaller than q^1, the unit price of the commodity z_i^1. This requires to reduce the levels of z_i^2 and z_i^0 (which is equivalent to reducing $c_i^{D_2}$ and c_i^I) and, even though the level of z_i^1 goes up due to the higher needs in that state of nature, it increases by less than the increase in L_{1i}, which means that the consumption level $c_i^{D_1}$ is also reduced. Again, the changes in $c_i^{D_1}$ and $c_i^{D_2}$ are the same, and the difference between these changes and the change in c_i^I reflects the change in the deductible faced by the individual. We show that the decreases in $c_i^{D_1}$ and $c_i^{D_2}$ are smaller than (resp. larger than and equal to) the decrease in c_i^I under DARA (resp. IARA and CARA). This means that the deductible is affected by L_{1i} in an opposite way than by w_i: an increase in L_{1i} decreases (resp. increases and does not affect) the deductible under DARA (resp. IARA and CARA) preferences. Indeed, since the resulting increase in y_i is not enough to offset the increase

[5] We can note that all the three commodities are normal goods since their levels increase when income goes up.

[6] DARA (resp. IARA and CARA) means that absolute risk aversion decreases (resp. increases and remains constant) when wealth increases. For more details, see Appendix A.

[7] See, for instance, Seog (2010). For the intuition of this result, note that a higher deductible means less insurance; thus, since under DARA (resp. IARA) wealthier people are less (resp. more) risk averse, they require less (resp. more) insurance.

in the expenditure for z_i^1, a rise in L_{1i} can be seen as an overall decrease in wealth, which explains the implications for the deductible under the different types of ARA. It can be easily understood that an increase in L_{2i} implies analogous results.

Turning to the loading cost, it first has to be noted that a change in λ affects the prices of the three commodities z_i^1, z_i^2 and z_i^0. In particular, an increase in λ increases q^1 and q^2 but decreases q^0. The impact of a rise in λ on the demands of the three commodities can thus be decomposed into the substitution and income effects. The substitution effect is negative for z_i^1 and z_i^2 and positive for z_i^0. Since, as explained above, we focus on the solutions with which $z_i^1 - z_i^0 > 0$ and $z_i^2 - z_i^0 > 0$, the income effect coming from the change in prices is negative: the levels of the commodities with increased prices are higher than the level of the commodity the price of which has decreased, so the individual becomes "poorer" and can thus afford lower levels of all commodities. There is, however, an additional income effect coming from the fact that a change in λ also affects labour supply and thus earnings y_i. We show in Appendix A that y_i is increasing in λ under DARA and CARA preferences, whereas the impact is undetermined under IARA. The total income effect, on the other hand, is shown to be negative under CARA and IARA but ambiguous under DARA. We thus have that $\frac{\partial z_i^1}{\partial \lambda}$ and $\frac{\partial z_i^2}{\partial \lambda}$ (which are in fact equal) are negative under CARA and IARA, but their sign is undetermined under DARA. The sign of $\frac{\partial z_i^0}{\partial \lambda}$ is always ambiguous.

Nevertheless, we can still say something about the difference between $\frac{\partial z_i^1}{\partial \lambda}$ or $\frac{\partial z_i^2}{\partial \lambda}$ and $\frac{\partial z_i^0}{\partial \lambda}$, which is equivalent to the difference between $\frac{\partial c_i^{D_1}}{\partial \lambda}$ or $\frac{\partial c_i^{D_2}}{\partial \lambda}$ and $\frac{\partial c_i^I}{\partial \lambda}$ and thus reflects the change in the deductible. In particular, we show that under IARA and CARA, $\frac{\partial c_i^{D_1}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} = \frac{\partial c_i^{D_2}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} < 0$ holds, which means that even though c_i^I decreases, it decreases by less than $c_i^{D_1}$ or $c_i^{D_2}$. The deductible thus increases with λ under IARA and CARA preferences. On the other hand, the sign of $\frac{\partial c_i^{D_1}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} = \frac{\partial c_i^{D_2}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda}$ and thus the effect on the deductible is ambiguous under DARA. To understand the intuition of these results, we should note that an increase in λ can also be interpreted as an increase in the price of insurance which can also be decomposed into the substitution and income effects. When λ goes up, the substitution effect pushes for buying less insurance (i.e. for a higher deductible), but the income effect has different consequences depending on ARA. First, it depends on ARA whether an increase in λ results in an overall increase or decrease in wealth (taking into account the reaction of y_i): there is a decrease under CARA and IARA and the effect is ambiguous under DARA. Under IARA preferences, the decrease in wealth causes a decrease in risk aversion and this, as the substitution effect, pushes for less insurance and so a higher deductible. Under CARA, the decrease in wealth has no impact on risk aversion and thus no impact on insurance deductible either. In that case, the income effect is equal to zero and the deductible increases simply due to the substitution effect. Under DARA, on the other hand, the deductible will certainly increase if there is an overall increase in wealth (i.e. if the increase in y_i is large enough). In that case, a higher wealth will imply a reduction in risk aversion and thus will push for less insurance. If, in contrast, there is an overall decrease in wealth,[8] risk aversion will go up and the income effect will

[8]This is always the case in the "standard" deductible insurance theory in which y_i is exogenous and is thus not affected

push for a lower deductible. The income effect will thus be opposite to the substitution effect and the total effect will be ambiguous. Therefore, if the income effect is large enough, under DARA preferences it is possible to have a situation where the demand for insurance increases (i.e. the deductible becomes lower) when its price goes up. Thus, as it is commonly recognized, under DARA preferences insurance might be a Giffen good.[9]

The main results of this section are summarized in Proposition 1.

Proposition 1. *As long as private insurance is associated with loading costs (i.e. $\lambda > 0$), the laissez-faire equilibrium insurance policy features a deductible. The equilibrium individual labour supply (and thus earnings) increases with the level of individual productivity and the level of LTC costs. Under DARA and CARA preferences, it also increases with the level of insurance loading costs, whereas under IARA preferences, the effect of loading costs is ambiguous. The equilibrium deductible is increasing (resp. decreasing and constant) in the level of individual productivity and decreasing (resp. increasing and constant) in the level of LTC costs under DARA (resp. IARA and CARA) preferences. Under IARA and CARA preferences, the deductible is increasing in the level of insurance loading costs, whereas under DARA preferences, the effect of loading costs is ambiguous.*

Concluding the discussion of the *laissez-faire*, it should be noted that obviously the *laissez-faire* choices are made separately by each type of individuals and there is thus no redistribution between the two types. One can however expect this situation to be suboptimal from the social point of view. Moreover, one can also expect the government to be able to provide insurance at a lower cost than private insurers, as it is the case with health insurance and pension schemes.[10] For these reasons, we now investigate what would be an optimal scheme of social LTC insurance.

3 Social insurance

We consider a utilitarian government which maximizes the sum of individual expected utilities.[11] We assume that insurance provision is not costless for the government, i.e. the government faces loading costs $\lambda^g > 0$ which reflect, for instance, the associated administrative expenses. However, we also allow for the fact that providing insurance might be less costly for the government than for private insurers, i.e. we consider $\lambda^g \leqslant \lambda$. We first study the first-best situation when the government has full information about the economy and then turn to the second-best scenario where the government cannot observe individual types. In particular, we assume that the government can observe the gross income y_i, the severity level of

by changes in λ. In our setting with endogenous labour supply, the possibility of an overall increase in wealth cannot be excluded if the increase in y_i is sufficiently large. The general conclusion that the effect of a change in λ on the insurance deductible is ambiguous under DARA remains nevertheless the same.

[9] See, for instance, Briys et al. (1989).

[10] Regarding the relative costs of private and public health insurance and pension schemes see Diamond (1992) and Mitchell (1998). Both argue that public costs tend to be lower than private ones. For the high loading costs in the private LTC insurance market, see Brown and Finkelstein (2007).

[11] Another possible approach would be to introduce Pareto weights on individual utilities, but this would make the analysis (in particular, the comparison of optimal deductibles faced by the two types of individuals) more cumbersome.

dependence (i.e. the state of nature), which can generally be objectively assessed according to specially designed scales such as, for instance, the Katz scale, and the levels of the three commodities z_i^1, z_i^2 and z_i^0 "consumed", but cannot observe individual productivity/wage, labour supply and the true LTC needs that a certain individual has at a given severity level. In that case, the government has to make sure that type h individuals will not mimic the individuals of type l. In other words, the government's problem then includes type h's incentive compatibility constraint.

As mentioned before, we consider both the case when the two types of individuals have the same LTC needs and the case when these needs differ.[12] For the latter case, we adopt a quite intuitive idea that more productive individuals might be somewhat more "spoiled" by their life, used to higher quality and more comfort or even feel obliged to comply with "standards" related to their social status, which might translate into their LTC needs being higher than those of the less productive type.[13] [14] Thus, for the case of different needs, we assume that, at each severity level of dependence, individuals of type h have higher LTC needs than individuals of type l (i.e. $L_{1h} > L_{1l}$ and $L_{2h} > L_{2l}$).[15] In this section, we are going to assume that the government recognizes these higher needs of type h as legitimate and thus accepts the fact that type h individuals need more. This is what we call a non-paternalistic case. On the other hand, the government might act in a paternalistic way in the sense of considering type h's higher needs as being whimsical and thus recognizing only a certain level of "legitimate" needs. We are going to study the paternalistic case separately in Section 4.

It should be also noted at this point that in the setting of type h having higher LTC needs than type l, it might be possible to have a *laissez-faire* outcome with type h being worse-off than type l, which, assuming that the government accepts all the needs, would require to redistribute resources from type l to type h. However, we focus on the (realistic) case where the needs of type h individuals are not too high and, due to their higher productivity, they still remain better-off in the *laissez-faire*.

We now state the general problem of the government and then we will analyze different scenarios such as the first-best and the second-best situations as well as the cases of identical and different individual LTC needs. The Lagrangean of the government's problem can thus be written as follows:

$$\mathcal{L} = \sum_{i=h,l} n_i \left[\pi_1 u\left(z_i^1 - L_{1i}\right) + \pi_2 u\left(z_i^2 - L_{2i}\right) + (1 - \pi_1 - \pi_2) u(z_i^0) - v\left(\frac{y_i}{w_i}\right)\right] +$$

$$+\mu \sum_{i=h,l} n_i \left[y_i - p^1 z_i^1 - p^2 z_i^2 - p^0 z_i^0 \right] +$$

[12] The dependence probabilities are assumed to remain the same for both types.

[13] For instance, these individuals might require more comfort or even "luxury" in a nursing home or want to go to a more "prestigious" nursing home.

[14] We have adopted this setting, namely uniform dependence probability with higher needs for the well to do for reasons of simplicity. If we had added the quite realistic idea that the dependence probability is higher for the unskilled than for the skilled, the analysis would have become much more intricate.

[15] Apart from assuming that h has higher needs than l in both dependence states of nature, we do not impose any structure on their need differences in the two states: we allow for $L_{1h} - L_{1l} \lesseqgtr L_{2h} - L_{2l}$ and discuss the implications of these different cases.

$$+\gamma[\pi_1 u\left(z_h^1 - L_{1h}\right) + \pi_2 u\left(z_h^2 - L_{2h}\right) + (1-\pi_1-\pi_2)u(z_h^0) - v\left(\frac{y_h}{w_h}\right) -$$

$$-\pi_1 u\left(z_l^1 - L_{1h}\right) - \pi_2 u\left(z_l^2 - L_{2h}\right) - (1-\pi_1-\pi_2)u(z_l^0) + v\left(\frac{y_l}{w_h}\right)] \tag{14}$$

where n_i is the share of type i ($i = h, l$) individuals in the society ($n_h + n_l = 1$), $p^1 \equiv (1+\lambda^g)\pi_1$, $p^2 \equiv (1+\lambda^g)\pi_2$ and $p^0 \equiv 1-(1+\lambda^g)\pi_1 - (1+\lambda^g)\pi_2$ are the prices of commodities z_i^1, z_i^2 and z_i^0 faced by the government whereas μ and γ are the Lagrange multipliers associated respectively with the resource and the incentive compatibility constraints. The incentive compatibility constraint ensures that type h individuals are better-off choosing their own allocation (i.e. z_h^1, z_h^2, z_h^0 and y_h) rather than the allocation of type l (i.e. z_l^1, z_l^2, z_l^0 and y_l).

For further use we also define $\widetilde{c}_l^{D_1} \equiv z_l^1 - L_{1h}$ and $\widetilde{c}_l^{D_2} \equiv z_l^2 - L_{2h}$ which are the wealth levels of dependent type h individuals who mimic the individuals of type l.

The FOCs for z_i^1, z_i^2, z_i^0 and y_i are given in Appendix B. We will now discuss their implications in different cases.

3.1 The first-best

To obtain the first-best problem, we simply need to set $\gamma = 0$ in the general specification. We can first note that the FOCs for y_h and y_l imply $\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} = \frac{v'\left(\frac{y_l}{w_l}\right)}{w_l}$, which means that $\frac{y_h}{w_h} > \frac{y_l}{w_l}$ and also $y_h > y_l$. The more productive type thus works (and earns) more than the less productive one. Moreover, we can immediately obtain $u'(c_h^{D_1}) = u'(c_h^{D_2}) = u'(c_l^{D_1}) = u'(c_l^{D_2})$, $u'(c_h^I) = u'(c_l^I)$ and

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)(1+\lambda^g)} < 1,\ i = h, l \tag{15}$$

In words, this means that wealth levels are equalized between the two individual types (i.e. there is a redistribution from type h to type l)[16] and between the two severity levels of dependence but are not equalized between the dependence states and the healthy state. More specifically, as long as $\lambda^g > 0$, it is optimal to provide less than full insurance for both types, which implies that the optimal allocation features a strictly positive deductible. Moreover, since wealth levels are equalized between the two types, it follows immediately that both types face the same deductible.[17]

Note that this is true both in the case when the two types have identical LTC needs and in the case when type h's needs are higher. The difference between the two cases is, however, reflected by the levels of z_i^1 and z_i^2. Take, for instance, the lower severity level of dependence. In that case, the equality of

[16] As in Mirrlees (1971), additive utility implies that in the first-best the more productive individual has a lower utility than the less productive one.

[17] Recall that the deductible can be defined as the difference $c_i^I - c_i^{D_1} = c_i^I - c_i^{D_2}$.

wealth levels implies $z_h^1 - L_{1h} = z_l^1 - L_{1l}$. If $L_{1h} = L_{1l}$, the levels of z^1 are also equalized between the two types. If, however, $L_{1h} > L_{1l}$, it must be that $z_h^1 > z_l^1$ also holds. This means that in the case of type h having higher needs, the redistribution from type h to type l is smaller. The same is true for the higher severity level.

This brings us to the question of how the first-best allocation can be decentralized in our economy. Let us first assume that the government faces the same loading costs as private insurers, i.e. $\lambda^g = \lambda$ (which, in turn, means that $p^j = q^j$, $j = 0, 1, 2$). In that case, it can be easily verified that the government's FOCs imply for both types exactly the same optimality conditions as the ones given by equations (11)-(13). The optimal and the *laissez-faire* tradeoffs thus coincide, meaning that there is no need to interfere with individual choices neither in terms of labour supply nor in terms of commodities z_i^1, z_i^2 and z_i^0 (or, alternatively, insurance purchases). Insurance provision can therefore be entirely left to the private market and the only role of the government is to redistribute wealth from type h to type l using lump-sum transfers. These transfers need to be lower when type h has higher LTC needs than type l. If, on the other hand, the government can provide insurance at a lower cost than private insurers (i.e. $\lambda^g < \lambda$), it is clearly more efficient to introduce social insurance than to rely on the private market.

The conclusions of this subsection can be summarized in the following proposition:

Proposition 2. *As long as providing insurance is costly for the government (i.e. $\lambda^g > 0$), the first-best optimal social LTC insurance features a deductible which is the same for both high and low productivity individuals. The first-best optimality also requires to equalize wealth between the two individual types in each of the three states of nature, but high productivity individuals are required to work more than low productivity ones. Social LTC insurance should be introduced if the government faces a lower loading cost than private insurers (i.e. $\lambda^g < \lambda$). If $\lambda^g = \lambda$, insurance can be left to the private market provided that lump-sum transfers from high to low productivity individuals are used by the government. These transfers are lower when high productivity individuals have higher LTC needs than low productivity ones.*

3.2 The second-best

We now come back to the general specification of the government's problem which includes the incentive constraint of type h. Let us begin by studying labour supply. Combining equations (56) and (57), we obtain the following optimality condition for type h:

$$\frac{v'\left(\frac{y_h}{w_h}\right)}{(1-\pi_1-\pi_2)u'(c_h^I)} = \frac{w_h}{p^0} \tag{16}$$

This is clearly the first-best tradeoff and, if $\lambda^g = \lambda$, it also coincides with the tradeoff obtained in the *laissez-faire*. Labour supply of type h is thus not distorted. On the other hand, combining (60) and (61), for type l we obtain

$$\frac{v'\left(\frac{y_l}{w_l}\right)}{(1-\pi_1-\pi_2)u'(c_l^I)} = \frac{w_l\left[n_l-\gamma\right]}{p^0\left[n_l-\gamma\frac{w_l v'\left(\frac{y_l}{w_h}\right)}{w_h v'\left(\frac{y_l}{w_l}\right)}\right]} < \frac{w_l}{p^0} \tag{17}$$

We thus see that labour supply of type l is distorted downwards, which helps to relax the incentive constraint of type h.

Turning to commodities z_i^1, z_i^2 and z_i^0, for type h we have

$$\frac{(1-\pi_1-\pi_2)u'(c_h^I)}{\pi_1 u'(c_h^{D_1})} = \frac{p^0}{p^1} \tag{18}$$

and

$$\frac{(1-\pi_1-\pi_2)u'(c_h^I)}{\pi_2 u'(c_h^{D_2})} = \frac{p^0}{p^2}, \tag{19}$$

which are also the first-best tradeoffs and coincide with the *laissez-faire* optimality conditions when $\lambda^g = \lambda$. Type h thus again faces no distortions. Note also that (18) and (19) imply $u'(c_h^{D_1}) = u'(c_h^{D_2})$ and can be rearranged to get equation (15) which shows us that the optimal allocation features a deductible as long as $\lambda^g > 0$.

For type l, on the other hand, we have the following tradeoffs:

$$\frac{(1-\pi_1-\pi_2)u'(c_l^I)}{\pi_1 u'(c_l^{D_1})} = \frac{p^0\left[n_l-\gamma\frac{u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_1})}\right]}{p^1\left[n_l-\gamma\right]} \tag{20}$$

and

$$\frac{(1-\pi_1-\pi_2)u'(c_l^I)}{\pi_2 u'(c_l^{D_2})} = \frac{p^0\left[n_l-\gamma\frac{u'(\tilde{c}_l^{D_2})}{u'(c_l^{D_2})}\right]}{p^2\left[n_l-\gamma\right]} \tag{21}$$

While the results so far were independent of whether the two types of individuals have the same or different LTC needs, the conclusions concerning the tradeoffs (20) and (21) and, consequently, the subsequent analysis are going to be substantially different in these two cases. We therefore study these cases separately.

3.2.1 Identical needs

Let us first note that if the needs of the two types are the same (i.e. $L_{1h} = L_{1l}$ and $L_{2h} = L_{2l}$), we have $\tilde{c}_l^{D_1} = c_l^{D_1}$ and $\tilde{c}_l^{D_2} = c_l^{D_2}$. This implies that (20) and (21) simply reduce to the first-best tradeoffs. Thus, just like type h, type l faces no distortions for commodities z_i^1, z_i^2 and z_i^0. Moreover, as for type h, it can be easily verified that (20) and (21) imply $u'(c_l^{D_1}) = u'(c_l^{D_2})$ and can be rearranged to get equation (15). Type l thus also faces a deductible as long as $\lambda^g > 0$.

We can then ask ourselves whether the deductibles faced by the two types are the same, as they were in the first-best. While it is not possible to compare D_h and D_l in the general case, it turns out using specific utility functions that the first-best result $D_h = D_l$ does not necessarily hold in the second-best. For instance, we show in Appendix C that we can have $D_h > D_l$ if the utility function is logarithmic. However, it is important to note that this result is only indirectly related to self-selection and redistribution. In fact, the reason for this result is that a logarithmic function is a function exhibiting DARA and that self-selection requires to leave some informational rent to type h, which means that type h remains wealthier than type l.[18] Under DARA, it is not surprising that a wealthier type faces a higher deductible. In contrast, we also show in Appendix C that if instead we assume an exponential utility function, which is a function exhibiting CARA, it becomes optimal to have $D_h = D_l$ as in the first-best. This implies that differences in the deductibles for the two types are due to risk aversion and not to distortions required by the second-best.

We can now discuss the implementation of the second-best in the case of identical needs. The first thing to note is that, unlike in the first-best, we now have a downward distortion of type l's labour supply, which implies a marginal tax on this type's income. On the other hand, as far as commodities z_i^1, z_i^2 and z_i^0 (or, equivalently, insurance coverage) are concerned, no distortions are required by the second-best. If private insurers have the same loading costs as the government, i.e. $\lambda = \lambda^g$, this means that the *laissez-faire* tradeoffs perfectly coincide with the optimal ones and there is thus no need for the government to tax or subsidize the three commodities (or, equivalently, insurance purchases). We find here the classical result of Atkinson and Stiglitz (1976) who show that, under separability between leisure and consumption, no commodity taxes are needed and the optimality can be achieved only through income taxation.[19] Thus, if $\lambda = \lambda^g$, the task of insurance can, as in the first-best, be entirely left to the private market. In contrast, if $\lambda > \lambda^g$, social insurance needs to be introduced since the government can provide insurance more efficiently than private insurers.

The case of identical needs can be summarized in the following proposition:

Proposition 3. *Assume that high and low productivity individuals have the same LTC needs. The second-best optimal allocation features a downward distortion of low productivity individuals' labour supply and an informational rent left to high productivity individuals, whereas insurance tradeoffs are not distorted. As long as providing insurance is costly for the government (i.e. $\lambda^g > 0$), the second-best*

[18]Indeed, we have

$$u'(c_h^{D_1}) = u'(c_h^{D_2}) = \frac{\mu(1+\lambda^g)n_h}{(n_h+\gamma)} < u'(c_l^{D_1}) = u'(c_l^{D_2}) = \frac{\mu(1+\lambda^g)n_l}{(n_l-\gamma)}$$

and

$$u'(c_h^I) = \frac{\mu n_h\left[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2\right]}{(1-\pi_1-\pi_2)\,(n_h+\gamma)} < u'(c_l^I) = \frac{\mu n_l\left[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2\right]}{(1-\pi_1-\pi_2)\,(n_l-\gamma)}.$$

[19]See also Stiglitz (1982).

optimal social insurance features a deductible which may be different for high and for low productivity individuals due to possibly different absolute risk aversion caused by incomplete redistribution between the two types. If the government faces a lower loading cost than private insurers (i.e. $\lambda^g < \lambda$), the implementation of the second-best optimum should rely on income-based social insurance with a marginal tax on low productivity individuals' income. If $\lambda^g = \lambda$, the second-best optimum can be implemented by introducing a non-linear income tax with a marginal tax on low productivity individuals' income and leaving insurance to the private market without any interference with individual choices.

3.2.2 Different needs

We now assume, as discussed above, that type h has higher LTC needs than type l (i.e. $L_{1h} > L_{1l}$ and $L_{2h} > L_{2l}$). In this case, the equalities $\widetilde{c}_l^{D_1} = c_l^{D_1}$ and $\widetilde{c}_l^{D_2} = c_l^{D_2}$ no longer hold and in particular, we have $\widetilde{c}_l^{D_1} < c_l^{D_1}$ and $\widetilde{c}_l^{D_2} < c_l^{D_2}$. Indeed, if type h wants to mimic type l, he has to "consume" the same amounts of commodities z^1 and z^2 as type l, but since he has higher needs, he is left with less wealth than type l. This implies $\frac{u'(\widetilde{c}_l^{D_1})}{u'(c_l^{D_1})} > 1$ and $\frac{u'(\widetilde{c}_l^{D_2})}{u'(c_l^{D_2})} > 1$, which, from equations (20) and (21), means that

$$\frac{(1-\pi_1-\pi_2)u'(c_l^I)}{\pi_1 u'(c_l^{D_1})} < \frac{p^0}{p^1} \tag{22}$$

and

$$\frac{(1-\pi_1-\pi_2)u'(c_l^I)}{\pi_2 u'(c_l^{D_2})} < \frac{p^0}{p^2} \tag{23}$$

Type l's "consumption" of commodities z^1 and z^2 is thus distorted downwards. To look at this in terms of insurance, we can rearrange equations (20) and (21) to get

$$\frac{u'(c_l^I)}{u'(c_l^{D_1})} = \frac{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}{(1-\pi_1-\pi_2)\,(1+\lambda^g)} \frac{\left[n_l - \gamma\frac{u'(\widetilde{c}_l^{D_1})}{u'(c_l^{D_1})}\right]}{[n_l-\gamma]} < \frac{1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)\,(1+\lambda^g)} \tag{24}$$

and

$$\frac{u'(c_l^I)}{u'(c_l^{D_2})} = \frac{1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)\,(1+\lambda^g)} \frac{\left[n_l - \gamma\frac{u'(\widetilde{c}_l^{D_2})}{u'(c_l^{D_2})}\right]}{[n_l-\gamma]} < \frac{1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)\,(1+\lambda^g)} \tag{25}$$

Thus, type l not only gets less than full insurance (i.e. a positive deductible) but also is given a worse coverage than in the first-best, i.e. his insurance is distorted downwards. This result is quite intuitive: since type h has higher needs, he values insurance more than type l; therefore, to make type l´s allocation less attractive it is optimal to distort his insurance downwards. It is also interesting to note that the ratios $\frac{u'(c_l^I)}{u'(c_l^{D_1})}$ and $\frac{u'(c_l^I)}{u'(c_l^{D_2})}$ would be smaller than 1 even with $\lambda^g = 0$, which means that type l would face a deductible even if the government had no loading costs.

Another important feature of this second-best setting is that the first-best equality $u'(c_l^{D_1}) = u'(c_l^{D_2})$ generally no longer holds. To see this, we can combine the FOC for z_l^1 (equation (58)) with the FOC for z_l^2 (equation (59)), which gives

$$n_l \left[u'(c_l^{D_1}) - u'(c_l^{D_2})\right] + \gamma \left[u'(\tilde{c}_l^{D_2}) - u'(\tilde{c}_l^{D_1})\right] = 0 \tag{26}$$

To analyze equation (26), let us first note that we can write the following:

$$\tilde{c}_l^{D_1} = z_l^1 - L_{1l} - \hat{L}_{1h} = c_l^{D_1} - \hat{L}_{1h}$$

and

$$\tilde{c}_l^{D_2} = z_l^2 - L_{2l} - \hat{L}_{2h} = c_l^{D_2} - \hat{L}_{2h}$$

where $\hat{L}_{1h} = L_{1h} - L_{1l} > 0$ and $\hat{L}_{2h} = L_{2h} - L_{2l} > 0$ are the differences between the needs of type h and type l. In other words, these are the additional needs that type h has to cover above the level of type l's needs.

Let us then evaluate equation (26) at the point where $c_l^{D_1} = c_l^{D_2}$. The first term will then disappear and the sign of the second term will obviously depend on the comparison between $\hat{L}_{1h}$ and $\hat{L}_{2h}$. Let us first assume that $\hat{L}_{2h} > \hat{L}_{1h}$, i.e. that the difference between the needs of type h and type l is larger when the severity level of dependence is high (state 2) than when it is low (state 1). In that case, the second term of (26) is positive, which means that z_l^1 (and, consequently, $c_l^{D_1}$) has to be increased. Thus, when $\hat{L}_{2h} > \hat{L}_{1h}$, we must have $c_l^{D_1} > c_l^{D_2}$. Obviously, the opposite holds when $\hat{L}_{2h} < \hat{L}_{1h}$. Only if $\hat{L}_{2h} = \hat{L}_{1h}$, we will have the equality $c_l^{D_1} = c_l^{D_2}$.

The intuition for this result is the following. Even if the wealth levels of type l were equalized ($c_l^{D_1} = c_l^{D_2}$), type h individuals who mimic type l would still face a disbalance, i.e. $\tilde{c}_l^{D_1}$ would not be equal to $\tilde{c}_l^{D_2}$, as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$. Obviously, mimickers would prefer type l's allocation to include a higher level of c_l in the state of nature where their additional needs are higher, which would allow them to achieve a better balance between the two states. However, to make type l's allocation less attractive to type h, type l´s allocation is designed exactly in the opposite way: in the state of nature where the additional needs of type h are higher, type l gets a lower level of wealth.

The fact that $c_l^{D_1}$ is generally not equal to $c_l^{D_2}$ also implies that type l no longer faces a state-independent deductible as it was the case before. Indeed, as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$, the deductibles faced by type l in the two dependence states of nature (defined as $D_{1l} = c_l^I - c_l^{D_1}$ and $D_{2l} = c_l^I - c_l^{D_2}$) are now different. In particular, type l now faces a higher deductible (i.e. less insurance) in the state of nature where the additional needs of type h are larger.[20]

[20] Interestingly, in this setting our results concerning type l are very close to the findings of Drèze and Schokkaert (2013) who study the relevance of Arrow's theorem under moral hazard. They also find a state-dependent deductible (the amount

As with identical needs, we can also ask ourselves how the optimal deductibles compare between the two types. This again depends on the specification of individual utility functions. The most informative case is that of an exponential utility function which, as mentioned above, exhibits CARA. It can be shown that with this utility function the state-independent deductible given to type h is lower than each of the state-dependent deductibles given to type l, i.e. $D_h < D_{1l}$ and $D_h < D_{2l}$. In this "pure" case in which absolute risk aversion does not depend on wealth, the comparison of the optimal deductibles exactly reflects the downward distortion of type l´s insurance. On the other hand, with different utility functions the influence of this distortion is less clearly seen because a role is also played by differences in absolute risk aversion caused by the differences in wealth present in the second-best.[21] For instance, with DARA preferences the comparison of the optimal deductibles between the two types is not clear since the lower wealth of type l pushes for a lower deductible for this type while the insurance distortion requires a higher one.

Finally, we can discuss how the above defined second-best optimum could be implemented. The main difference compared to the case of identical needs is that now type l faces distortions not only in terms of labour supply but also in terms of commodities z^1 and z^2 (or, equivalently, insurance coverage). Indeed, even when $\lambda^g = \lambda$, the optimal tradeoffs (20) and (21) now differ from the *laissez-faire* ones and, more specifically, imply that type l's "consumption" of z^1 and z^2 has to be taxed. We thus see that the result of Atkinson and Stiglitz (1976) no longer holds. This is not surprising given that in this setting individuals differ in more than one unobservable characteristic[22] (i.e. they differ not only in productivity but also in LTC needs).

Looking at the problem in terms of insurance, we also see that type l's insurance is distorted downwards and, in addition to this, there is generally a distortion between the two dependence states of nature. This clearly suggests that, even if $\lambda^g = \lambda$, the government now needs to interfere with individual insurance choices. To better understand this interference, in Appendix D we provide a more explicit analysis of the second-best implementation through private insurance. In particular, we come back to the initial specification of the individual problem (as presented in Section 1) in which individuals earn income y_i, pay private insurance premiums P_i and get insurance benefits $\alpha_{1i}L_{1i}$ and $\alpha_{2i}L_{2i}$. We assume that the government can observe all these variables and consider (non-linear) policy instruments which are based on them. We first show that all marginal tax rates are zero for type h and that type l's income is taxed at the margin. We then turn to type l's insurance and consider the possibility to tax his insurance premium as well as insurance benefits received in each of the two dependence states. We show that the three instruments can be chosen in several ways (also by setting one of them to zero) but that at least two of them are needed unless $\hat{L}_{2h} = \hat{L}_{1h}$. In other words, taxing only the premium is generally not enough: it is possible only if $\widehat{L}_{2h} = \widehat{L}_{1h}$, which is the case when there is no need to distort insurance between

of which depends on the price elasticity) and show that a deductible is optimal even when loading costs are zero.

[21] Using the FOCs in Appendix B it can be verified that type h has lower marginal utilities than type l in all states of nature, i.e. is again given informational rent.

[22] See, for instance, Cremer et al. (2001).

the dependence states of nature. Otherwise, in addition to the marginal tax on the premium, a marginal tax or subsidy is needed on the insurance benefits received in at least one of the two dependence states. Alternatively, one can have a zero marginal tax on the premium and tax, at different rates, the insurance benefits received at both severity levels of dependence. Note also that instead of taxing or subsidizing insurance benefits, one could tax or subsidize the deductibles paid by the individuals. In fact, taxing insurance benefits has the same effect as subsidizing the deductible: a tax on insurance benefits forces individuals to reduce their insurance (i.e. to increase their deductibles) while a subsidy on the deductible also encourages to choose higher deductibles (and thus less insurance).

With the above instruments in place, the task of insurance can again be left to the private market if $\lambda^g = \lambda$. If $\lambda^g < \lambda$, it is more efficient to introduce social insurance.

We now summarize the above derived results in the following proposition:

Proposition 4. *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government recognizes all needs as legitimate. The second-best optimal allocation features an informational rent left to high productivity individuals and a downward distortion of low productivity individuals' labour supply as well as of their insurance coverage. Moreover, if the difference between the needs of high and low productivity individuals is not the same at both severity levels of dependence (i.e. $\hat{L}_{2h} \neq \hat{L}_{1h}$), low productivity individuals also face a distortion of insurance tradeoff between the two severity levels. Optimal social LTC insurance features a deductible for high productivity individuals as long as providing insurance is costly for the government (i.e. $\lambda^g > 0$), whereas low productivity individuals face a deductible even when $\lambda^g = 0$. High productivity individuals face a state-independent deductible, while the deductible for low productivity individuals is state-dependent as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$. If the government faces a lower loading cost than private insurers (i.e. $\lambda^g < \lambda$), the implementation of the second-best optimum should rely on income-based social insurance with a marginal tax on low productivity individuals' income. If $\lambda^g = \lambda$, private insurance can be involved, but this requires certain instruments aimed at low productivity individuals' insurance purchases. If $\widehat{L}_{2h} = \widehat{L}_{1h}$, these instruments can be limited to a marginal tax on their insurance premiums. Otherwise, one also needs a marginal tax/subsidy on their insurance benefits (or their deductibles) in at least one of the two dependence states. A marginal tax on low productivity individuals' income is also required.*

4 The case of paternalism

As discussed above, we now turn to the idea that fully recognizing the higher needs of the somewhat "spoiled" type h might be an inappropriate approach for the government. In this section we thus assume that the government recognizes as legitimate only a certain level of needs: $\bar{L}_1$ when the severity level of dependence is low and $\bar{L}_2 > \bar{L}_1$ when the severity level is high. For simplicity, we assume that the legitimate levels of needs coincide with the needs of type l, i.e. $\bar{L}_1 = L_{1l} < L_{1h}$ and $\bar{L}_2 = L_{2l} < L_{2h}$.

Since the government considers the needs $\bar{L}_1$ and $\bar{L}_2$ as sufficient, only these needs are taken into account in its objective function. The government's objective function thus writes as

$$\sum_{i=h,l} n_i \left[\pi_1 u\left(\bar{c}_i^{D_1}\right) + \pi_2 u\left(\bar{c}_i^{D_2}\right) + (1-\pi_1-\pi_2) u(c_i^I) - v\left(\frac{y_i}{w_i}\right) \right]$$

where $\bar{c}_i^{D_1} = z_i^1 - \bar{L}_1$ and $\bar{c}_i^{D_2} = z_i^2 - \bar{L}_2$. The bar above the wealth levels denotes the fact that the government considers only $\bar{L}_1$ and $\bar{L}_2$. Note that for type l, $\bar{c}_l^{D_1} = c_l^{D_1}$ and $\bar{c}_l^{D_2} = c_l^{D_2}$, but for type h, $\bar{c}_h^{D_1} > c_h^{D_1} = z_h^1 - L_{1h}$ and $\bar{c}_h^{D_2} > c_h^{D_2} = z_h^2 - L_{2h}$.

Apart from the objective function, the problem of the government writes in the same way as in the previous section. The FOCs are also the same except that in the FOCs for z_i^1 and z_i^2, the terms coming from the objective function contain wealth levels $\bar{c}_i^{D_1}$ and $\bar{c}_i^{D_2}$ rather than $c_i^{D_1}$ and $c_i^{D_2}$. For type l this implies exactly the same FOCs as in the previous section, whereas for type h the FOCs for z_h^1 and z_h^2 now write as

$$\frac{\partial \mathcal{L}}{\partial z_h^1} = n_h \pi_1 u'(\bar{c}_h^{D_1}) - \mu n_h p^1 + \gamma \pi_1 u'(c_h^{D_1}) = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial z_h^2} = n_h \pi_2 u'(\bar{c}_h^{D_2}) - \mu n_h p^2 + \gamma \pi_2 u'(c_h^{D_2}) = 0 \tag{28}$$

As in the previous section, we first discuss the first-best setting with full information (in which $\gamma = 0$) and then look at the second-best with unobservable types.

4.1 The first-best

Since the FOCs for y_h and y_l are the same as in the previous section, the results concerning labour supply also remain the same. As far as wealth levels are concerned, we now have $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_h^{D_2}) = u'(\bar{c}_l^{D_1}) = u'(\bar{c}_l^{D_2})$ and $u'(c_h^I) = u'(c_l^I)$. Thus, from the paternalistic point of view (i.e. taking into account only the legitimate needs), there is an equality between the two types and between the two severity levels of dependence. Moreover, we have

$$\frac{u'(c_i^I)}{u'(\bar{c}_i^{D_1})} = \frac{u'(c_i^I)}{u'(\bar{c}_i^{D_2})} = \frac{1 - \pi_1(1+\lambda^g) - \pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)(1+\lambda^g)} < 1, \; i = h, l \tag{29}$$

which implies a strictly positive deductible as long as $\lambda^g > 0$. From the paternalistic point of view, this deductible is the same for both types. However, note that the situation is different in terms of the true levels of individual wealth. Since $\bar{c}_h^{D_1} > c_h^{D_1}$ and $\bar{c}_h^{D_2} > c_h^{D_2}$, in the two dependence states of nature type h effectively has lower wealth levels than type l, which also translates into type h effectively facing a higher deductible than type l. Comparing to the case of no paternalism where the government equalizes the *true* levels of individual wealth and thus requires less redistribution from h to l when h has higher needs, we see that here type h is no longer given any "compensation" for the fact that he needs to spend more and he therefore ends up with a lower wealth than type l.

Similarly, looking at the tradeoffs between the true marginal utilities in different states of nature, we see that type l faces exactly the same tradeoffs as in the first-best with no paternalism, whereas for type h these tradeoffs are different. In particular, we have

$$\frac{u'(c_h^I)}{u'(c_h^{D_j})} < \frac{1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)}{(1-\pi_1-\pi_2)\,(1+\lambda^g)},\; j=1,2 \tag{30}$$

which shows that now type h is not insured against his LTC needs as well as before. Indeed, since his needs are now higher than accepted by the government, a part of his needs is not taken into account in the determination of socially optimal insurance, which results in him being insured against his true needs more "poorly" than before. In addition to this, note that generally type h no longer has equal marginal utilities in the two dependence states of nature. In particular, $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_h^{D_2})$ implies

$$\frac{u'(c_h^{D_1})}{u'(c_h^{D_2})} = \frac{u'(\bar{c}_h^{D_1} - \hat{L}_{1h})}{u'(\bar{c}_h^{D_2} - \hat{L}_{2h})} \gtreqless 1 \; if \; \hat{L}_{1h} \gtreqless \hat{L}_{2h} \tag{31}$$

where $\hat{L}_{1h}$ and $\hat{L}_{2h}$ are defined as before as the differences between the needs of type h and type l which are now also equivalent to the differences between the needs of type h and the legitimate needs. Indeed, since the government does not take into account a part of type h's needs, the socially optimal insurance does not properly balance his wealth in the two dependence states of nature if the parts of the needs which are not accounted for are different in these two states, as it can be seen in (31). This means that, if $\hat{L}_{2h} \neq \hat{L}_{1h}$, type h effectively faces state-dependent deductibles.

The discussion above implies that the decentralization of the first-best optimum requires no interference with the choices of type l but does require some "correction" of those of type h. Indeed, if $\lambda^g = \lambda$, the first-best allocation implies that for type h we must have

$$\frac{(1-\pi_1-\pi_2)u'(c_h^I)}{\pi_1 u'(c_h^{D_1})} < \frac{q^0}{q^1}, \tag{32}$$

and

$$\frac{(1-\pi_1-\pi_2)u'(c_h^I)}{\pi_2 u'(c_h^{D_2})} < \frac{q^0}{q^2}, \tag{33}$$

which means that type h's "consumption" of commodities z^1 and z^2 has to be taxed. Reasoning in terms of insurance, (30) implies that we need to tax type h's private insurance purchases.[23] Since the government does not recognize the full needs of type h, from its point of view, type h buys too much

[23] The decentralization in terms of insurance follows the same reasoning as shown explicitly in Appendix D for the case of type l in the second-best without paternalism.

insurance and thus a tax is needed to "correct" these purchases. However, taxing only type h's premium is generally not enough since, as can be seen from (31), the first-best allocation implies that type h´s marginal utilities in the two dependence states of nature are not equalized as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$. This requires an additional tax or subsidy applied to the private insurance benefits (or the deductible) in one of the dependence states. Indeed, the policy has to correct for the fact that type h takes into account "unnecessary" needs which exceed the sufficient (legitimate) needs and so a different extent of correction is needed in the states of nature where the legitimate needs are exceeded by different amounts. This means that type h is forced to buy insurance with state-dependent deductibles. In addition to these corrections, lump-sum transfers need to be used to redistribute resources from h to l.

If $\lambda^g < \lambda$, the decentralization should not rely on private but rather on social insurance. In this setting, social insurance would be based on the legitimate needs and the government would cover the costs above the deductible only until the level of these legitimate needs (and not until the level of his true needs for type h). Both types would face the same state-independent *social insurance* deductible, but type h's *effectively faced* deductible would be higher and generally state-dependent: in addition to the social insurance deductible, he would have to pay $\hat{L}_{1h}$ in state 1 and $\hat{L}_{2h}$ in state 2, which means that he would pay more than type l and that the total amount paid in the two states would be different as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$.

The paternalistic first-best can be summarized in the following proposition:

Proposition 5. *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government does not accept these higher needs as legitimate. As long as providing insurance is costly for the government (i.e. $\lambda^g > 0$), the first-best optimal social LTC insurance features a deductible which is the same for both types of individuals. However, high productivity individuals effectively face higher and, as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$, state-dependent deductibles since their higher needs are not taken into account by the government. If the government faces a lower loading cost than private insurers (i.e. $\lambda^g < \lambda$), the decentralization of the first-best optimum should rely on social LTC insurance. If $\lambda^g = \lambda$, private insurance can be involved, but this requires certain corrective instruments aimed at high productivity individuals' insurance purchases. If $\widehat{L}_{2h} = \widehat{L}_{1h}$, these instruments can be limited to a marginal tax on their insurance premiums. Otherwise, one also needs a marginal tax/subsidy on their insurance benefits (or their deductibles) in at least one of the two dependence states. Lump-sum transfers from high to low productivity individuals are also needed.*

4.2 The second-best

Let us first note that for type l, we have the same results as in the second-best with different needs and no paternalism. The paternalistic case, however, implies differences for type h.

Looking at type h, first, using (56), (27) and (28), we get (for $j = 1, 2$)

$$\frac{u'(c_h^I)}{u'(\bar{c}_h^{D_j})} = \frac{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}{(1-\pi_1-\pi_2)(1+\lambda^g)} \frac{\left[n_h + \gamma \frac{u'(c_h^{D_j})}{u'(\bar{c}_h^{D_j})}\right]}{[n_h+\gamma]} > \frac{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}{(1-\pi_1-\pi_2)(1+\lambda^g)} \tag{34}$$

Comparing this to (29), we see that, from the paternalistic point of view, there is an upward distortion of type h's insurance compared to the first-best allocation. In other words, there is a better insurance coverage against the legitimate needs than in the first-best. This comes from the need to ensure type h's incentive compatibility: even though social insurance is based only on the legitimate needs, to prevent mimicking the government makes a concession by providing a more generous coverage against these needs than in the first-best. Thus, while type h still has additional needs which are not covered at all, he is at least better covered against the legitimate needs. Note also that for type l, insurance against the legitimate needs (which coincide with his true needs) is distorted downwards (see equations (24) and (25)).

If, on the other hand, we look at the true marginal utilities faced by type h, we can see that the better coverage provided against the legitimate needs is still not sufficient to restore the tradeoffs obtained for type h without paternalism. In particular, we have (for $j = 1, 2$)

$$\frac{u'(c_h^I)}{u'(c_h^{D_j})} = \frac{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}{(1-\pi_1-\pi_2)(1+\lambda^g)} \frac{\left[n_h \frac{u'(\bar{c}_h^{D_j})}{u'(c_h^{D_j})} + \gamma\right]}{[n_h+\gamma]} < \frac{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}{(1-\pi_1-\pi_2)(1+\lambda^g)} \tag{35}$$

Therefore, in terms of type h´s true marginal utilities, there is still a downward distortion of his insurance coverage due to the presence of paternalism. Nevertheless, as noted above, type l´s insurance is also distorted downwards.

In addition to this, we generally no longer have the paternalistic first-best equality $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_h^{D_2})$. To see this, let us combine (27) with (28), which gives

$$n_h \left[u'(\bar{c}_h^{D_1}) - u'(\bar{c}_h^{D_2})\right] + \gamma \left[u'(c_h^{D_1}) - u'(c_h^{D_2})\right] = 0 \tag{36}$$

Noting that we can write $c_h^{D_1} = \bar{c}_h^{D_1} - \hat{L}_{1h}$ and $c_h^{D_2} = \bar{c}_h^{D_2} - \hat{L}_{2h}$, let us evaluate equation (36) at the point where $\bar{c}_h^{D_1} = \bar{c}_h^{D_2}$. The first term will then disappear and the sign of the second term will depend on the comparison between $\hat{L}_{1h}$ and $\hat{L}_{2h}$. Let us first assume that $\hat{L}_{2h} > \hat{L}_{1h}$. In that case, the second term of (36) is negative, which means that z_h^1 (and, consequently, $\bar{c}_h^{D_1}$) has to be decreased. Thus, when $\hat{L}_{2h} > \hat{L}_{1h}$, we must have $\bar{c}_h^{D_1} < \bar{c}_h^{D_2}$. The opposite holds when $\hat{L}_{2h} < \hat{L}_{1h}$. Only if $\hat{L}_{2h} = \hat{L}_{1h}$, we will have the equality $\bar{c}_h^{D_1} = \bar{c}_h^{D_2}$.

Recall that for type l, the equality $\bar{c}_l^{D_1} = \bar{c}_l^{D_2}$ (which is equivalent to $c_l^{D_1} = c_l^{D_2}$) does not generally hold either (see the discussion after equation (26)). However, for type h, the comparison of wealth levels (from the paternalistic point of view) in the two dependence states of nature is exactly opposite to their comparison for type l. Indeed, in contrast to type l, type h is given a higher "paternalistic" level of wealth in the state of nature where the difference between his true needs and the legitimate needs is higher. The intuition for this result is quite simple. Since the paternalistic government recognizes only the legitimate needs, its first-best solution, as we have seen above, is to equalize the "paternalistic" wealth levels in the two dependence states. For type h, this means that his true wealth levels are not equalized as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$. In the second-best, however, the government has to ensure incentive compatibility and so it makes a certain concession in the sense of granting type h a better (although still not perfect) balance of his true wealth levels, which is achieved by giving him more wealth in the state of nature where his additional needs are higher.

Note that this also implies that, unlike in the first-best, the "paternalistic" deductible (i.e. the deductible which is optimal when only the legitimate needs are accepted) for type h is now state-dependent (and defined as $D_{1h} = c_h^I - \bar{c}_h^{D_1}$ and $D_{2h} = c_h^I - \bar{c}_h^{D_2}$): in the state of nature where his additional needs are higher, type h faces a lower deductible (and thus more insurance against the legitimate needs). His effectively faced deductibles are also state-dependent since his true wealth levels, although better balanced, are still not equalized.

It can also be verified that the second-best setting again implies some informational rent given to type h. In particular, we have that $u'(c_h^I) < u'(c_l^I)$, $u'(\bar{c}_h^{D_1}) < u'(\bar{c}_l^{D_1})$ and $u'(\bar{c}_h^{D_2}) < u'(\bar{c}_l^{D_2})$ hold. It should be noted that if we consider the true marginal utilities of type h in the two dependence states of nature, the comparison between the two types becomes less clear and it is not ruled out that type h can still have a lower wealth than type l because of his additional needs; however, type h is now given some advantage compared to the first-best allocation where we had $u'(c_h^I) = u'(c_l^I)$, $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_l^{D_1})$ and $u'(\bar{c}_h^{D_2}) = u'(\bar{c}_l^{D_2})$.

As in the previous cases, we can also discuss the comparison of optimal deductibles between the two types. This comparison again requires to use specific utility functions and is again the most informative in the case of exponential utility exhibiting CARA. However, even with CARA, in this case we can have an unambiguous comparison only of the "paternalistic" deductibles. With CARA, it can be shown that type h faces a lower "paternalistic" deductible than type l in both dependence states of nature, i.e. $D_{1h} < D_{1l}$ and $D_{2h} < D_{2l}$. This reflects the above derived result that the second-best requires to provide a better insurance against the legitimate needs to type h than to type l. As discussed before, CARA utility allows to isolate this consideration since it is not influenced by differences in wealth. On the other hand, the comparison of the effectively faced deductibles is less obvious. Since type h has additional needs, his effectively faced deductible is higher than the "paternalistic" one and it is not clear whether it is still lower than the deductible faced by type l. In contrast, with DARA preferences, even the comparison of the "paternalistic" deductibles is not evident since, similarly to the case of no paternalism, wealth differences then come into play as well.

Finally, let us look at how the second-best allocation can be implemented. As before, if $\lambda^g = \lambda$, the implementation can involve private insurance, but now we need interference with the choices of both individual types. Insurance of both types has to be taxed at the margin, but for different reasons: type l's insurance is distorted to ensure self-selection, whereas type h faces a paternalistic correction. As discussed in the previous cases, if $\hat{L}_{2h} \neq \hat{L}_{1h}$, taxing only insurance premiums is not sufficient and thus additional marginal taxes or subsidies need to be applied to both types' private insurance benefits (or their private insurance deductibles) in at least one of the dependence states of nature. In addition to this, a non-linear income tax with a marginal tax on type l's income is also needed.

On the other hand, if $\lambda^g < \lambda$, social insurance should be introduced. As in the first-best, this insurance would be based on the legitimate needs, but, as long as $\hat{L}_{2h} \neq \hat{L}_{1h}$, social insurance deductibles would now be state-dependent: type h's deductibles would be designed as the "paternalistic" deductibles discussed above and type l's deductibles would be designed in the same way as in the non-paternalistic case.

The paternalistic second-best is summarized in Proposition 6:

Proposition 6. *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government does not accept these higher needs as legitimate. The second-best optimal allocation has the following features:*

a) Low productivity individuals face a downward distortion of their labour supply and insurance coverage. Moreover, if $\hat{L}_{2h} \neq \hat{L}_{1h}$, they also face a distortion of the insurance tradeoff between the two severity levels of dependence.

b) As in the first-best, high productivity individuals face paternalistic corrections of their insurance coverage, but the paternalism is now "softer": there is a better balance of their true wealth levels in the two states of dependence and a better coverage against the legitimate needs. Moreover, high productivity individuals get informational rent.

If the government faces a lower loading cost than private insurers (i.e. $\lambda^g < \lambda$), the implementation of the second-best optimum should rely on income-based social LTC insurance with a marginal tax on low productivity individuals' income. As long as $\hat{L}_{2h} \neq \hat{L}_{1h}$, optimal social insurance features state-dependent deductibles for both individual types. If $\lambda^g = \lambda$, private insurance can be involved, but this requires certain instruments aimed at both individual types' insurance purchases. If $\widehat{L}_{2h} = \widehat{L}_{1h}$, these instruments can be limited to marginal taxes on their insurance premiums. Otherwise, one also needs marginal taxes/subsidies on their insurance benefits (or their deductibles) in at least one of the two dependence states. A marginal tax on low productivity individuals' income is also required.

Conclusion

In this paper, we have studied the design of an optimal social LTC insurance which would address the growing concerns of many (especially middle class) people that LTC costs might force them to spend down all their wealth. Recent suggestions made by Dilnot's Commission (2011) in the UK raise the idea of capping individual LTC spending. While this idea is very much in the spirit of Arrow's (1963) theorem of the deductible, we were interested in exploring more formally whether this well-known result of (private) insurance theory can be applied to social LTC insurance and how such a social policy should be designed. To do this, we considered a model in which two types of individuals, skilled and unskilled, face the risk of becoming dependent, and their dependence can have a low or a high degree of severity. We first looked at the individual choices in the *laissez-faire* and then investigated optimal social insurance under different scenarios. In particular, we studied separately the case where, at each severity level of dependence, both types of individuals have the same LTC needs and the case where these needs are higher for high productivity (skilled) individuals. In the latter case, we considered two different positions that could be taken by the government: a non-paternalistic scenario where the government recognizes all needs as legitimate and a paternalistic case where the government does not accept the "whims" of high productivity individuals. In all the cases, we first looked at the first-best setting with full information and then considered the second-best situation when the government cannot observe individual types.

Our results show that, as long as providing insurance is not costless for the government, optimal social LTC insurance indeed features a deductible. In the first-best setting when the government has full information about individual types, it is optimal to give the same deductible to both types of individuals because wealth is perfectly equalized between the two types. In the second-best, the situation is somewhat different due to the presence of self-selection constraints. Moreover, the influence of self-selection constraints is also rather different depending on whether the two types of individuals have the same or different LTC needs. With identical needs, the second-best optimality does not require any distortions of insurance tradeoffs. In fact, if in that case loading costs of private and social insurance are the same and if optimal non-linear income taxation is introduced, the government can leave the task of insurance to the private market without any need to interfere with individual choices, which is in line with the classical result of Atkinson and Stiglitz (1976). The absence of insurance distortions, however, does not necessarily mean that optimal deductibles will be the same for both individual types: due to asymmetric information, the redistribution of resources is incomplete and thus wealth differences remain, which implies different absolute risk aversion for the two types of individuals under DARA or IARA preferences. This in turn results in different deductibles being optimal for the two types. Nevertheless, equal deductibles remain optimal under CARA.

Insurance distortions, however, come into play when skilled individuals have higher LTC needs than the unskilled. In that case, self-selection requires to distort downwards the insurance coverage of unskilled individuals, which among other things means that they will face a positive deductible even if insurance is costless for the government. Moreover, if the difference between the needs of skilled and unskilled individuals is not the same at both severity levels of dependence, unskilled individuals also face a distortion of the insurance tradeoff between the two severity levels, which again helps to make their allocation less attractive to the skilled. In other words, this means that generally it becomes optimal to give the unskilled state-dependent deductibles rather than a unique one as before. This constitutes a departure from a straightforward application of Arrow's theorem, even though it still remains optimal to have a deductible at each severity level.

These distortions for the unskilled apply in both the paternalistic and the non-paternalistic case. On the other hand, skilled individuals face no distortions in the non-paternalistic case but this is no longer true in the paternalistic one. In the paternalistic case, there is a mismatch between socially optimal and the skilled type's individual tradeoffs already in the first-best because the government considers different needs than skilled individuals do. In that case, one has to make a distinction between social insurance explicitly provided by the government (and based on the legitimate needs) and the "true" level of insurance that is implied for skilled individuals who have additional needs which they must fully cover themselves. Indeed, even though in the first-best social insurance features the same deductible for both types of individuals, skilled individuals effectively pay higher amounts which are equal to the social insurance deductible plus their additional costs. Moreover, if the additional costs are not the same at both severity levels, skilled individuals effectively face state-dependent deductibles even though the explicit social insurance deductible is state-independent. Consequently, if the first-best outcome is to be decentralized using private insurance, "corrections" of skilled individuals' choices are needed because in the private market they want to buy too much insurance from the social point of view.

The need for paternalistic corrections remains in the second-best as well; however, the presence of the self-selection constraint forces the government to "soften" its paternalism. Social insurance becomes more generous in the sense that it provides a better coverage against the legitimate needs than in the first-best (and than the coverage provided to unskilled individuals). Moreover, if the difference between the needs of skilled individuals and the legitimate needs is not the same at both severity levels of dependence, it becomes optimal to have state-dependent social insurance deductibles for skilled individuals too. The idea is to allow skilled individuals to achieve a better balance between their wealth levels in the two dependence states as these levels are not equalized because of differences in uncovered additional costs.

While there is a number of differences between the paternalistic and the non-paternalistic case, the comparison of the second-best social insurance deductibles between the two individual types has a similar pattern in both cases. The downward distortion of unskilled individuals' insurance coverage present in both cases and complemented in the paternalistic case by the upward distortion of skilled individuals' coverage against the legitimate needs implies that at each severity level, the skilled face lower social insurance deductibles than the unskilled under CARA preferences. The equality obtained with identical needs is thus no longer valid. On the other hand, while the case of CARA constitutes a useful benchmark, it nevertheless is likely to be a rather unrealistic assumption. Therefore, differences in absolute risk aversion are expected to play a role as well, which makes the influence of insurance distortions less clear and thus the comparison of optimal deductibles less obvious.

References

[1] Arrow K. (1963). "Uncertainty and the welfare economics of medical care". *American Economic Review*, 53, 941-973.

[2] Atkinson, A. B. and J. E. Stiglitz (1976). "The Design of Tax Structure: Direct versus Indirect Taxation". *Journal of Public Economics*, 6 (1-2), 55-75.

[3] Briys, E., G. Dionne and L. Eeckhoudt (1989). "More on Insurance as a Giffen Good". *Journal of Risk and Uncertainty*, 2, 4, 415-420.

[4] Brown, J. and A. Finkelstein (2007). "Why is the Market for Long-Term Care Insurance So Small?". *Journal of Public Economics*, 91, 1967-1991.

[5] Cremer, H., J-M. Lozachmeur and P. Pestieau (2016). "The design of long term care insurance contracts". *Journal of Health Economics*, http://dx.doi.org/10.1016/j.jhealeco.2016.08.008

[6] Cremer, H., P. Pestieau and G. Ponthière (2012). "The Economics of Long-Term-Care: A Survey". *Nordic Economic Policy Review*, 2, 107-148.

[7] Cremer, H., P. Pestieau and J-C. Rochet (2001). "Direct versus indirect taxation: the design of the tax structure revisited". *International Economic Review*, 42, 3, 781-799.

[8] Diamond, P. A. (1992). "Organizing the Health Insurance Market". *Econometrica*, 60, 1233-1254.

[9] Dilnot Commission (2011). "Fairer care funding: The report of the Commission on Funding of Care and Support". London: Commission on Funding of Care and Support.

[10] Drèze, J. and E. Schokkaert (2013). "Arrow's theorem of the deductible: Moral hazard and stop-loss in health insurance". *Journal of Risk and Uncertainty*, 47, 2, 147-163.

[11] Drèze, J., P. Pestieau and E. Schokkaert (2016). "Arrow's theorem of the deductible and long-term care insurance". *Economics Letters*, http://dx.doi.org/10.1016/j.econlet.2016.08.042

[12] Kessler, D. (2008). "The Long-Term Care Insurance Market". *The Geneva Papers*, 33, 33-40.

[13] Mitchell, O. (1998). "Administrative Costs in Public and Private Retirement Systems", in M. Feldstein (ed.), *Privatizing Social Security*. University of Chicago Press, Chicago.

[14] Norton, E. (2000). "Long-Term Care", in Cuyler, A. and J. Newhouse (eds.), *Handbook of Health Economics*, Volume 1b, Chapter 17.

[15] Seog, S.H. (2010). *"The economics of risk and insurance"*. Wiley-Blackwell.

[16] Stiglitz, J. E. (1982). "Self-Selection and Pareto Efficient Taxation". *Journal of Public Economics*, 17, 213-240.

Appendix A: comparative statics in the *laissez-faire*

Fully differentiating equations (8), (9) and (10) with respect to w_i, we get respectively

$$\frac{\partial z_i^1}{\partial w_i}\left[(q^0)^2\pi_1 u''(c_i^{D_1}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^1)^2\right] -$$

$$-\frac{\partial y_i}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1 + \frac{\partial z_i^2}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 = 0, \tag{37}$$

$$\frac{\partial z_i^2}{\partial w_i}\left[(q^0)^2\pi_2 u''(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^2)^2\right] -$$

$$-\frac{\partial y_i}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^2 + \frac{\partial z_i^1}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 = 0 \tag{38}$$

and

$$\frac{\partial y_i}{\partial w_i}\left[(1-\pi_1-\pi_2)\,u''(c_i^I) - (q^0)^2\frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2}\right] -$$

$$-\frac{\partial z_i^1}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1 - \frac{\partial z_i^2}{\partial w_i}(1-\pi_1-\pi_2)\,u''(c_i^I)q^2 +$$

$$+(q^0)^2\frac{v''\left(\frac{y_i}{w_i}\right)y_i}{w_i^3} + (q^0)^2\frac{v'\left(\frac{y_i}{w_i}\right)}{w_i^2} = 0 \tag{39}$$

Solving the system of equations (37), (38) and (39) for $\frac{\partial z_i^1}{\partial w_i}$, $\frac{\partial z_i^2}{\partial w_i}$ and $\frac{\partial y_i}{\partial w_i}$, we obtain

$$\frac{\partial y_i}{\partial w_i} = \frac{[1]\cdot[2]}{[3]} > 0,$$

$$\frac{\partial z_i^1}{\partial w_i} = \frac{(1-\pi_1-\pi_2)\,u''(c_i^I)q^1\pi_2 u''(c_i^{D_2})\frac{\partial y_i}{\partial w_i}}{[1]} > 0,$$

$$\frac{\partial z_i^2}{\partial w_i} = \frac{(1-\pi_1-\pi_2)\,u''(c_i^I)q^2\pi_1 u''(c_i^{D_1})\frac{\partial y_i}{\partial w_i}}{[1]} > 0$$

where

$$[1] \equiv \pi_2 u''(c_i^{D_2})(q^0)^2 \pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})(1-\pi_1-\pi_2) u''(c_i^I)(q^1)^2 + (1-\pi_1-\pi_2) u''(c_i^I)(q^2)^2 \pi_1 u''(c_i^{D_1}) > 0,$$

$$[2] \equiv -(q^0)^2 \frac{v''\left(\frac{y_i}{w_i}\right) y_i}{w_i^3} - (q^0)^2 \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i^2} < 0,$$

$$[3] \equiv (1-\pi_1-\pi_2) u''(c_i^I) \pi_2 u''(c_i^{D_2})(q^0)^2 \pi_1 u''(c_i^{D_1}) - (q^0)^2 \frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2} [1] < 0.$$

Using equation (7), we have

$$\frac{\partial z_i^0}{\partial w_i} = \frac{1}{q^0}\left[\frac{\partial y_i}{\partial w_i} - q^1 \frac{\partial z_i^1}{\partial w_i} - q^2 \frac{\partial z_i^2}{\partial w_i}\right] = \frac{q^0 \pi_2 u''(c_i^{D_2}) \pi_1 u''(c_i^{D_1}) \frac{\partial y_i}{\partial w_i}}{[1]} > 0.$$

Since from equations (8) and (9) we have $u'(c_i^{D_1}) = u'(c_i^{D_2}) \Leftrightarrow c_i^{D_1} = c_i^{D_2}$, it is also true that $u''(c_i^{D_1}) = u''(c_i^{D_2})$. Using this and the definitions of q^1 and q^2, it is straightforward to see that $\frac{\partial z_i^1}{\partial w_i} = \frac{\partial z_i^2}{\partial w_i}$. Moreover, note that $\frac{\partial z_i^0}{\partial w_i} = \frac{\partial c_i^I}{\partial w_i}$ and, since L_{1i} and L_{2i} remain unchanged, we also have that $\frac{\partial z_i^1}{\partial w_i} = \frac{\partial c_i^{D_1}}{\partial w_i}$ and $\frac{\partial z_i^2}{\partial w_i} = \frac{\partial c_i^{D_2}}{\partial w_i}$.

We can thus write

$$\frac{\partial c_i^{D_1}}{\partial w_i} - \frac{\partial c_i^I}{\partial w_i} = \frac{\partial c_i^{D_2}}{\partial w_i} - \frac{\partial c_i^I}{\partial w_i} = \frac{\frac{\partial y_i}{\partial w_i} \pi_2 u''(c_i^{D_2}) [4]}{[1]} \tag{40}$$

where

$$[4] \equiv (1-\pi_1-\pi_2)(1+\lambda)\pi_1 u''(c_i^I) - [1-(1+\lambda)(\pi_1+\pi_2)] \pi_1 u''(c_i^{D_1}).$$

The sign of [4] is ambiguous in the general case and differs depending on the absolute risk aversion (ARA) exhibited by the utility function. In particular, we are now going to show that $[4] > 0$ under decreasing absolute risk aversion (DARA), $[4] < 0$ under increasing absolute risk aversion (IARA) and $[4] = 0$ under constant absolute risk aversion (CARA).

To see this, let us first note that DARA (resp. IARA and CARA) means that

$$ARA(c) = \frac{-u''(c)}{u'(c)} < (\text{resp.} > \text{ and } =)\ ARA(d) = \frac{-u''(d)}{u'(d)} \quad \text{for } c > d,$$

where $\frac{-u''(x)}{u'(x)}$ is the Arrow-Pratt measure of absolute risk aversion at wealth x.

Thus, noting that from (8) we have $c_i^I > c_i^{D_1}$, under DARA (resp. IARA and CARA) preferences we can write

$$\frac{-u''(c_i^I)}{u'(c_i^I)} < (\text{resp.} > \text{ and } =) \ \frac{-u''(c_i^{D_1})}{u'(c_i^{D_1})}$$

$$\Longleftrightarrow$$

$$u''(c_i^I) > (\text{resp.} < \text{ and } =) \ \frac{u''(c_i^{D_1})}{u'(c_i^{D_1})} \ u'(c_i^I)$$

We can then multiply both sides by $(1-\pi_1-\pi_2)\,(1+\lambda)\pi_1$ and subtract $[1-(1+\lambda)(\pi_1+\pi_2)]\,\pi_1 u''(c_i^{D_1})$ from both sides, which gives

$$(1-\pi_1-\pi_2)\,(1+\lambda)\pi_1 u''(c_i^I) - [1-(1+\lambda)(\pi_1+\pi_2)]\,\pi_1 u''(c_i^{D_1})$$

$$> (\text{resp.} < \text{ and } =) \ \frac{u''(c_i^{D_1})}{u'(c_i^{D_1})} \left[\ u'(c_i^I)\,(1-\pi_1-\pi_2)\,(1+\lambda)\pi_1 - [1-(1+\lambda)(\pi_1+\pi_2)]\,\pi_1 u'(c_i^{D_1}) \right] = 0 \tag{41}$$

noting that the expression in the last big bracket is zero from equation (8).

The left-hand side of inequality (41) is exactly the definition of [4]; we therefore indeed have that under DARA (resp. IARA and CARA), [4] $>$ (resp. $<$ and $=$) 0. We can now use this in (40), which gives that

$$\frac{\partial c_i^{D_1}}{\partial w_i} - \frac{\partial c_i^I}{\partial w_i} = \frac{\partial c_i^{D_2}}{\partial w_i} - \frac{\partial c_i^I}{\partial w_i} < (\text{resp.} > \text{ and } =) \ 0$$

under DARA (resp. IARA and CARA) preferences.

Fully differentiating equations (8), (9) and (10) with respect to L_{1i}, we get respectively

$$\frac{\partial z_i^1}{\partial L_{1i}} \left[(q^0)^2 \pi_1 u''(c_i^{D_1}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^1)^2\right] - \frac{\partial y_i}{\partial L_{1i}}\,(1-\pi_1-\pi_2)\,u''(c_i^I)q^1 +$$

$$+\frac{\partial z_i^2}{\partial L_{1i}}\,(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 - (q^0)^2\pi_1 u''(c_i^{D_1}) = 0, \tag{42}$$

$$\frac{\partial z_i^2}{\partial L_{1i}} \left[(q^0)^2 \pi_2 u''(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^2)^2\right] -$$

$$-\frac{\partial y_i}{\partial L_{1i}}\,(1-\pi_1-\pi_2)\,u''(c_i^I)q^2 + \frac{\partial z_i^1}{\partial L_{1i}}\,(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 = 0 \tag{43}$$

and

$$\frac{\partial y_i}{\partial L_{1i}} \left[(1-\pi_1-\pi_2)\,u''(c_i^I) - (q^0)^2 \frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2}\right] -$$

$$-\frac{\partial z_i^1}{\partial L_{1i}}\left(1-\pi_1-\pi_2\right)u''(c_i^I)q^1-\frac{\partial z_i^2}{\partial L_{1i}}\left(1-\pi_1-\pi_2\right)u''(c_i^I)q^2=0 \tag{44}$$

Solving the system of equations (42), (43) and (44) for $\frac{\partial z_i^1}{\partial L_{1i}}$, $\frac{\partial z_i^2}{\partial L_{1i}}$ and $\frac{\partial y_i}{\partial L_{1i}}$, we obtain

$$\frac{\partial y_i}{\partial L_{1i}}=\frac{q^1\left(1-\pi_1-\pi_2\right)u''(c_i^I)(q^0)^2\pi_1 u''(c_i^{D_1})\pi_2 u''(c_i^{D_2})}{[3]}>0, \tag{45}$$

$$\frac{\partial z_i^1}{\partial L_{1i}}=\frac{\left(1-\pi_1-\pi_2\right)u''(c_i^I)q^1\pi_2 u''(c_i^{D_2})\frac{\partial y_i}{\partial L_{1i}}}{[1]}+$$

$$+\frac{\pi_1 u''(c_i^{D_1})\left[(q^0)^2\pi_2 u''(c_i^{D_2})+(q^2)^2\left(1-\pi_1-\pi_2\right)u''(c_i^I)\right]}{[1]}>0,$$

$$\frac{\partial z_i^2}{\partial L_{1i}}=\frac{\left(1-\pi_1-\pi_2\right)u''(c_i^I)q^2\pi_1 u''(c_i^{D_1})\left[\frac{\partial y_i}{\partial L_{1i}}-q^1\right]}{[1]}<0.$$

The last sign follows from the fact that $\frac{\partial y_i}{\partial L_{1i}}<q^1$. This can be easily seen from (45) using the above definition of [3] and noting that

$$\frac{\left(1-\pi_1-\pi_2\right)u''(c_i^I)(q^0)^2\pi_1 u''(c_i^{D_1})\pi_2 u''(c_i^{D_2})}{[3]}<1.$$

Using equation (7), we have

$$\frac{\partial z_i^0}{\partial L_{1i}}=\frac{1}{q^0}\left[\frac{\partial y_i}{\partial L_{1i}}-q^1\frac{\partial z_i^1}{\partial L_{1i}}-q^2\frac{\partial z_i^2}{\partial L_{1i}}\right]=\frac{q^0\pi_2 u''(c_i^{D_2})\pi_1 u''(c_i^{D_1})\left[\frac{\partial y_i}{\partial L_{1i}}-q^1\right]}{[1]}<0.$$

Note that $\frac{\partial z_i^0}{\partial L_{1i}}=\frac{\partial c_i^I}{\partial L_{1i}}$ and, since L_{2i} remains unchanged, $\frac{\partial z_i^2}{\partial L_{1i}}=\frac{\partial c_i^{D_2}}{\partial L_{1i}}$. On the other hand, we have

$$\frac{\partial c_i^{D_1}}{\partial L_{1i}}=\frac{\partial z_i^1}{\partial L_{1i}}-1=\frac{\left(1-\pi_1-\pi_2\right)u''(c_i^I)q^1\pi_2 u''(c_i^{D_2})\left[\frac{\partial y_i}{\partial L_{1i}}-q^1\right]}{[1]}<0.$$

Using the fact that $u''(c_i^{D_1})=u''(c_i^{D_2})$ and the definitions of q^1 and q^2, it is straightforward to see that $\frac{\partial c_i^{D_1}}{\partial L_{1i}}=\frac{\partial c_i^{D_2}}{\partial L_{1i}}$.

We can thus write

$$\frac{\partial c_i^{D_1}}{\partial L_{1i}}-\frac{\partial c_i^I}{\partial L_{1i}}=\frac{\partial c_i^{D_2}}{\partial L_{1i}}-\frac{\partial c_i^I}{\partial L_{1i}}=\frac{\left[\frac{\partial y_i}{\partial L_{1i}}-q^1\right]\pi_2 u''(c_i^{D_2})\,[4]}{[1]} \tag{46}$$

Recalling from above that [4] > (resp. < and =) 0 under DARA (resp. IARA and CARA) preferences, we have that

$$\frac{\partial c_i^{D_1}}{\partial L_{1i}} - \frac{\partial c_i^I}{\partial L_{1i}} = \frac{\partial c_i^{D_2}}{\partial L_{1i}} - \frac{\partial c_i^I}{\partial L_{1i}} > (\text{resp.} < \text{ and } =)\ 0$$

under DARA (resp. IARA and CARA).

Fully differentiating equations (8), (9) and (10) with respect to λ, we get respectively

$$\frac{\partial z_i^1}{\partial \lambda}\left[(q^0)^2\pi_1 u''(c_i^{D_1}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^1)^2\right] - \frac{\partial y_i}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1 +$$

$$+\frac{\partial z_i^2}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 - q^0\pi_1 u'(c_i^{D_1})(\pi_1+\pi_2) - q^0\pi_1(1-\pi_1-\pi_2)\,u'(c_i^I) +$$

$$+(1-\pi_1-\pi_2)\,u''(c_i^I)q^1\left[\pi_1\left(z_i^1 - z_i^0\right) + \pi_2\left(z_i^2 - z_i^0\right)\right] = 0, \tag{47}$$

$$\frac{\partial z_i^2}{\partial \lambda}\left[(q^0)^2\pi_2 u''(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u''(c_i^I)(q^2)^2\right] - \frac{\partial y_i}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^2 +$$

$$+\frac{\partial z_i^1}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1q^2 - q^0\pi_2 u'(c_i^{D_2})(\pi_1+\pi_2) - q^0\pi_2(1-\pi_1-\pi_2)\,u'(c_i^I) +$$

$$+(1-\pi_1-\pi_2)\,u''(c_i^I)q^2\left[\pi_1\left(z_i^1 - z_i^0\right) + \pi_2\left(z_i^2 - z_i^0\right)\right] = 0 \tag{48}$$

and

$$\frac{\partial y_i}{\partial \lambda}\left[(1-\pi_1-\pi_2)\,u''(c_i^I) - (q^0)^2\frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2}\right] - \frac{\partial z_i^1}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^1 -$$

$$-\frac{\partial z_i^2}{\partial \lambda}(1-\pi_1-\pi_2)\,u''(c_i^I)q^2 + q^0(\pi_1+\pi_2)\frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} -$$

$$-(1-\pi_1-\pi_2)\,u''(c_i^I)\left[\pi_1\left(z_i^1 - z_i^0\right) + \pi_2\left(z_i^2 - z_i^0\right)\right] = 0 \tag{49}$$

Solving the system of equations (47), (48) and (49) for $\frac{\partial z_i^1}{\partial \lambda}$, $\frac{\partial z_i^2}{\partial \lambda}$ and $\frac{\partial y_i}{\partial \lambda}$, we obtain

$$\frac{\partial y_i}{\partial \lambda} = \frac{(1-\pi_1-\pi_2)\,u''(c_i^I)(q^0)^2\pi_1 u''(c_i^{D_1})\pi_2 u''(c_i^{D_2})\left[\pi_1\left(z_i^1 - z_i^0\right) + \pi_2\left(z_i^2 - z_i^0\right)\right]}{[3]} +$$

$$+\frac{(1-\pi_1-\pi_2)\,u'(c_i^I)q^0\pi_2 u''(c_i^{D_2})(\pi_1+\pi_2)\,[4]}{[3]}, \tag{50}$$

$$\frac{\partial z_i^1}{\partial \lambda} = \frac{\left(1-\pi_1-\pi_2\right) u''(c_i^I) q^1 \pi_2 u''(c_i^{D_2}) \left[\frac{\partial y_i}{\partial \lambda} - \pi_1 \left(z_i^1 - z_i^0\right) - \pi_2 \left(z_i^2 - z_i^0\right)\right]}{[1]} +$$

$$+ \frac{q^0 \pi_2 u''(c_i^{D_2}) \pi_1 \left[u'(c_i^{D_1}) \left(\pi_1+\pi_2\right) + \left(1-\pi_1-\pi_2\right) u'(c_i^I)\right]}{[1]}, \tag{51}$$

$$\frac{\partial z_i^2}{\partial \lambda} = \frac{\left(1-\pi_1-\pi_2\right) u''(c_i^I) q^2 \pi_1 u''(c_i^{D_1}) \left[\frac{\partial y_i}{\partial \lambda} - \pi_1 \left(z_i^1 - z_i^0\right) - \pi_2 \left(z_i^2 - z_i^0\right)\right]}{[1]} +$$

$$+ \frac{q^0 \pi_1 u''(c_i^{D_1}) \pi_2 \left[u'(c_i^{D_2}) \left(\pi_1+\pi_2\right) + \left(1-\pi_1-\pi_2\right) u'(c_i^I)\right]}{[1]} \tag{52}$$

Let us first discuss $\frac{\partial y_i}{\partial \lambda}$. It is easy to see that its first term is always positive, while the sign of the second term depends on the sign of [4]. Therefore, the second term is positive (resp. negative and equal to zero) under DARA (resp. IARA and CARA) preferences. This implies that $\frac{\partial y_i}{\partial \lambda}$ is clearly positive under DARA and CARA, but its sign is ambiguous under IARA. Moreover, it can be easily seen that $\frac{(1-\pi_1-\pi_2)u''(c_i^I)(q^0)^2\pi_1 u''(c_i^{D_1})\pi_2 u''(c_i^{D_2})}{[3]} < 1$. This means that the first term of $\frac{\partial y_i}{\partial \lambda}$ is smaller than $\pi_1 \left(z_i^1 - z_i^0\right) + \pi_2 \left(z_i^2 - z_i^0\right)$. We therefore have $\frac{\partial y_i}{\partial \lambda} < \pi_1 \left(z_i^1 - z_i^0\right) + \pi_2 \left(z_i^2 - z_i^0\right)$ under CARA and IARA preferences, while this comparison is ambiguous under DARA.

Let us now turn to $\frac{\partial z_i^1}{\partial \lambda}$ and $\frac{\partial z_i^2}{\partial \lambda}$. First, using the fact that $u''(c_i^{D_1}) = u''(c_i^{D_2})$ and the definitions of q^1 and q^2, it is straightforward to see that $\frac{\partial z_i^1}{\partial \lambda} = \frac{\partial z_i^2}{\partial \lambda}$. It can then be noted that the second terms of $\frac{\partial z_i^1}{\partial \lambda}$ and $\frac{\partial z_i^2}{\partial \lambda}$ are always negative (substitution effects as explained in the main text). As for the first terms (income effects), it follows from the above discussion of $\frac{\partial y_i}{\partial \lambda}$ that they are also negative under CARA and IARA, but their sign is ambiguous under DARA. Thus, under CARA and IARA we clearly have $\frac{\partial z_i^1}{\partial \lambda} = \frac{\partial z_i^2}{\partial \lambda} < 0$, while under DARA the sign is undetermined.

Using equation (7), we have

$$\frac{\partial z_i^0}{\partial \lambda} = \frac{1}{q^0}\left[\frac{\partial y_i}{\partial \lambda} - q^1 \frac{\partial z_i^1}{\partial \lambda} - q^2 \frac{\partial z_i^2}{\partial \lambda}\right] = \frac{q^0 \pi_2 u''(c_i^{D_2}) \pi_1 u''(c_i^{D_1}) \left[\frac{\partial y_i}{\partial \lambda} - \pi_1 \left(z_i^1 - z_i^0\right) - \pi_2 \left(z_i^2 - z_i^0\right)\right]}{[1]} -$$

$$- \frac{\pi_2 u''(c_i^{D_2}) \pi_1 \left(q^1 + q^2\right) \left[u'(c_i^{D_1}) \left(\pi_1+\pi_2\right) + \left(1-\pi_1-\pi_2\right) u'(c_i^I)\right]}{[1]}$$

The sign of $\frac{\partial z_i^0}{\partial \lambda}$ is ambiguous. While its second term is always positive (substitution effect), the ambiguity comes from the first term (income effect). Under CARA and IARA preferences the first term is negative, which makes the sign of the whole expression undetermined. Under DARA preferences the sign of the first term is itself undetermined.

Note that $\frac{\partial z_i^0}{\partial \lambda} = \frac{\partial c_i^I}{\partial \lambda}$ and, since L_{1i} and L_{2i} remain unchanged, we have that $\frac{\partial z_i^1}{\partial \lambda} = \frac{\partial c_i^{D_1}}{\partial \lambda}$ and $\frac{\partial z_i^2}{\partial \lambda} = \frac{\partial c_i^{D_2}}{\partial \lambda}$.

We can thus write

$$\frac{\partial c_i^{D_1}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} = \frac{\partial c_i^{D_2}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} = \frac{\left[\frac{\partial y_i}{\partial \lambda} - \pi_1\left(z_i^1 - z_i^0\right) - \pi_2\left(z_i^2 - z_i^0\right)\right]\pi_2 u''(c_i^{D_2})[4]}{[1]} +$$

$$+\frac{\pi_2 u''(c_i^{D_2})\pi_1\left[u'(c_i^{D_1})\left(\pi_1+\pi_2\right) + \left(1-\pi_1-\pi_2\right)u'(c_i^I)\right]}{[1]} \tag{53}$$

The second term of the RHS of (53) (substitution effect) is always negative. The first term (income effect) is also negative under IARA preferences but is equal to zero under CARA and undetermined under DARA. We thus have that

$$\frac{\partial c_i^{D_1}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} = \frac{\partial c_i^{D_2}}{\partial \lambda} - \frac{\partial c_i^I}{\partial \lambda} < 0$$

under IARA and CARA, but the sign is undetermined under DARA.

Appendix B: FOCs of the government's problem (general case)

The FOCs of the government's problem in the general case write as follows:

$$\frac{\partial \mathcal{L}}{\partial z_h^1} = \pi_1 u'(c_h^{D_1})\left[n_h + \gamma\right] - \mu n_h p^1 = 0 \tag{54}$$

$$\frac{\partial \mathcal{L}}{\partial z_h^2} = \pi_2 u'(c_h^{D_2})\left[n_h + \gamma\right] - \mu n_h p^2 = 0 \tag{55}$$

$$\frac{\partial \mathcal{L}}{\partial z_h^0} = (1-\pi_1-\pi_2)u'(c_h^I)\left[n_h + \gamma\right] - \mu n_h p^0 = 0 \tag{56}$$

$$\frac{\partial \mathcal{L}}{\partial y_h} = -v'\left(\frac{y_h}{w_h}\right)\frac{1}{w_h}\left[n_h + \gamma\right] + \mu n_h = 0 \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial z_l^1} = n_l \pi_1 u'(c_l^{D_1}) - \mu n_l p^1 - \gamma \pi_1 u'(\widetilde{c}_l^{D_1}) = 0 \tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial z_l^2} = n_l \pi_2 u'(c_l^{D_2}) - \mu n_l p^2 - \gamma \pi_2 u'(\widetilde{c}_l^{D_2}) = 0 \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial z_l^0} = (1-\pi_1-\pi_2)u'(c_l^I)\left[n_l-\gamma\right] - \mu n_l p^0 = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial y_l} = -n_l v'\left(\frac{y_l}{w_l}\right)\frac{1}{w_l} + \mu n_l + \gamma v'\left(\frac{y_l}{w_h}\right)\frac{1}{w_h} = 0 \tag{61}$$

Appendix C: specific examples with identical needs

Recall that $D_h = c_h^I - c_h^{D_1} = c_h^I - c_h^{D_2}$ and $D_l = c_l^I - c_l^{D_1} = c_l^I - c_l^{D_2}$.

Let us first assume that $u(x) = lnx$. Then, from (54) and (55) we have $c_h^{D_1} = c_h^{D_2} = \frac{(n_h+\gamma)}{\mu(1+\lambda^g)n_h}$ and from (56), $c_h^I = \frac{(1-\pi_1-\pi_2)(n_h+\gamma)}{\mu n_h[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2]}$.

Similarly, from (58) and (59) we have $c_l^{D_1} = c_l^{D_2} = \frac{(n_l-\gamma)}{\mu(1+\lambda^g)n_l}$ and from (60), $c_l^I = \frac{(1-\pi_1-\pi_2)(n_l-\gamma)}{\mu n_l[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2]}$.

We then get

$$D_h = \frac{(n_h+\gamma)\lambda^g}{\mu n_h\left[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2\right](1+\lambda^g)}$$

and

$$D_l = \frac{(n_l-\gamma)\lambda^g}{\mu n_l\left[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2\right](1+\lambda^g)}.$$

Noting that $\frac{(n_h+\gamma)}{n_h} > 1$ and $\frac{(n_l-\gamma)}{n_l} < 1$, we have $D_h > D_l$.

Let us now consider $u(x) = -e^{-x}$. Now, from (54) and (55) we have $c_h^{D_1} = c_h^{D_2} = -ln\left[\frac{\mu(1+\lambda^g)n_h}{(n_h+\gamma)}\right]$ and from (56), $c_h^I = -ln\left[\frac{\mu n_h[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2]}{(1-\pi_1-\pi_2)(n_h+\gamma)}\right]$.

Similarly, from (58) and (59) we have $c_l^{D_1} = c_l^{D_2} = -ln\left[\frac{\mu(1+\lambda^g)n_l}{(n_l-\gamma)}\right]$ and from (60) we have $c_l^I = -ln\left[\frac{\mu n_l[1-(1+\lambda^g)\pi_1-(1+\lambda^g)\pi_2]}{(1-\pi_1-\pi_2)(n_l-\gamma)}\right]$.

We then obtain

$$D_h = D_l = ln\left[\frac{(1+\lambda^g)(1-\pi_1-\pi_2)}{[1-\pi_1(1+\lambda^g)-\pi_2(1+\lambda^g)]}\right].$$

Appendix D: implementation of the (non-paternalistic) second-best with different needs

In this appendix, we come back to the initial specification of the individual problem with an explicit modeling of private insurance (as presented in Section 1). Individuals thus earn income y_i, pay private insurance premiums P_i and get insurance benefits $\alpha_{1i}L_{1i}$ and $\alpha_{2i}L_{2i}$. We assume that the government can observe all these variables and consider the following (non-linear) policy instruments:

- Tax based on income and insurance premiums $T(y_i, P_i)$ paid before the realisation of the state of nature;
- Tax based on insurance benefits in the state of low severity dependence $T_1(\alpha_{1i}L_{1i})$;
- Tax based on insurance benefits in the state of high severity dependence $T_2(\alpha_{2i}L_{2i})$.

We also assume here that the government has the same loading costs as private insurers, i.e. $\lambda^g = \lambda$.

Given the government's policy, the Lagrangean of individual i writes as

$$\mathcal{L} = \pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1-\pi_1-\pi_2)\,u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right) - \\ + \mu_i\left[P_i - \pi_1(1+\lambda)\alpha_{1i}L_{1i} - \pi_2(1+\lambda)\alpha_{2i}L_{2i}\right]$$

where
$c_i^{D_1} = y_i - P_i - T(y_i, P_i) - (1-\alpha_{1i})L_{1i} - T_1(\alpha_{1i}L_{1i})$
$c_i^{D_2} = y_i - P_i - T(y_i, P_i) - (1-\alpha_{2i})L_{2i} - T_2(\alpha_{2i}L_{2i})$
$c_i^I = y_i - P_i - T(y_i, P_i)$.

Assuming interior solutions, the individual FOCs write as follows:

$$\frac{\partial\mathcal{L}}{\partial y_i} = \left[\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u'(c_i^I)\right](1-T_{y_i}) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{62}$$

$$\frac{\partial\mathcal{L}}{\partial P_i} = \left[\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u'(c_i^I)\right](-1-T_{p_i}) + \mu_i = 0 \tag{63}$$

$$\frac{\partial\mathcal{L}}{\partial \alpha_{1i}} = u'(c_i^{D_1})\left[1 - T_1'(\alpha_{1i}L_{1i})\right] - \mu_i(1+\lambda) = 0 \tag{64}$$

$$\frac{\partial\mathcal{L}}{\partial \alpha_{2i}} = u'(c_i^{D_2})\left[1 - T_2'(\alpha_{2i}L_{2i})\right] - \mu_i(1+\lambda) = 0 \tag{65}$$

where T_{y_i} and T_{p_i} denote the partial derivatives of T with respect to y_i and P_i.

From (62), we have

$$\frac{v'\left(\frac{y_i}{w_i}\right)}{(1-\pi_1-\pi_2)u'(c_i^I)} = w_i\,(1-T_{y_i})\left[1 + \frac{\pi_1 u'(c_i^{D_1})}{(1-\pi_1-\pi_2)\,u'(c_i^I)} + \frac{\pi_2 u'(c_i^{D_2})}{(1-\pi_1-\pi_2)\,u'(c_i^I)}\right] \tag{66}$$

Combining (64) and (65), we get

$$\frac{u'(c_i^{D_1})}{u'(c_i^{D_2})} = \frac{\left[1 - T_2'(\alpha_{2i}L_{2i})\right]}{\left[1 - T_1'(\alpha_{1i}L_{1i})\right]} \tag{67}$$

Combining (64) and (63), we can get

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{-\pi_1}{(1-\pi_1-\pi_2)} + \frac{[1-T_1'(\alpha_{1i}L_{1i})]}{(1-\pi_1-\pi_2)(1+\lambda)(1+T_{p_i})} - \frac{\pi_2}{(1-\pi_1-\pi_2)}\frac{[1-T_1'(\alpha_{1i}L_{1i})]}{[1-T_2'(\alpha_{2i}L_{2i})]} \tag{68}$$

Finally, combining (65) and (63), we can get

$$\frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{-\pi_2}{(1-\pi_1-\pi_2)} + \frac{[1-T_2'(\alpha_{2i}L_{2i})]}{(1-\pi_1-\pi_2)(1+\lambda)(1+T_{p_i})} - \frac{\pi_1}{(1-\pi_1-\pi_2)}\frac{[1-T_2'(\alpha_{2i}L_{2i})]}{[1-T_1'(\alpha_{1i}L_{1i})]} \tag{69}$$

Let us first look at labour supply. For type h, combining (66) with (16), (18) and (19), we have $T_{y_h} = 0$.

For type l, combining (66) with (17), (20) and (21), we get

$$T_{y_l} = 1 - \frac{[n_l - \gamma]\,\beta}{\left[n_l - \gamma \frac{w_l v'\left(\frac{y_l}{w_h}\right)}{w_h v'\left(\frac{y_l}{w_l}\right)}\right]} > 0$$

where

$$\beta = \frac{\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_1})}\right]\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_2})}{u'(c_l^{D_2})}\right]}{\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_1})}\right]\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_2})}{u'(c_l^{D_2})}\right] + p^1\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_2})}{u'(c_l^{D_2})}\right]\left[\gamma\frac{u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_1})} - \gamma\right] + p^2\left[n_l - \gamma\frac{u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_1})}\right]\left[\gamma\frac{u'(\tilde{c}_l^{D_2})}{u'(c_l^{D_2})} - \gamma\right]} < 1.$$

Turning to insurance, from the discussion in Subsection 3.2 it follows immediately that for type h we have $T_{p_h} = 0$, $T_1'(\alpha_{1h}L_{1h}) = 0$ and $T_2'(\alpha_{2h}L_{2h}) = 0$.

For type l, combining the government's FOCs (58) and (59), we can get

$$\frac{u'(c_i^{D_1})}{u'(c_i^{D_2})} = 1 - \frac{\gamma}{n_l}\left[\frac{u'(\tilde{c}_l^{D_2}) - u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_2})}\right].$$

Combining this with (67), we have

$$\frac{[1-T_2'(\alpha_{2i}L_{2i})]}{[1-T_1'(\alpha_{1i}L_{1i})]} = 1 - \frac{\gamma}{n_l}\left[\frac{u'(\tilde{c}_l^{D_2}) - u'(\tilde{c}_l^{D_1})}{u'(c_l^{D_2})}\right] \tag{70}$$

We know from the discussion in the main text that the optimal allocation implies $u'(c_i^{D_1}) < u'(c_i^{D_2})$ if $\widehat{L}_{2h} > \widehat{L}_{1h}$, $u'(c_i^{D_1}) > u'(c_i^{D_2})$ if $\widehat{L}_{2h} < \widehat{L}_{1h}$ and $u'(c_i^{D_1}) = u'(c_i^{D_2})$ if $\widehat{L}_{2h} = \widehat{L}_{1h}$. We thus have that $T_2'(\alpha_{2l}L_{2l}) > T_1'(\alpha_{1l}L_{1l})$ if $\widehat{L}_{2h} > \widehat{L}_{1h}$, $T_2'(\alpha_{2l}L_{2l}) < T_1'(\alpha_{1l}L_{1l})$ if $\widehat{L}_{2h} < \widehat{L}_{1h}$ and $T_2'(\alpha_{2l}L_{2l}) = T_1'(\alpha_{1l}L_{1l})$ if $\widehat{L}_{2h} = \widehat{L}_{1h}$.

Further, combining (68) with (24), using (70) and rearranging, we obtain

$$\frac{[1 - T_1'(\alpha_{1l} L_{1l})]}{(1 + T_{p_l})} = \frac{[1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)] \left[n_l - \gamma \frac{u'(\widetilde{c}_l^{D_1})}{u'(c_l^{D_1})} \right]}{[n_l - \gamma]} +$$

$$+ \pi_1(1+\lambda) + \frac{\pi_2(1+\lambda) n_l u'(c_l^{D_2})}{n_l u'(c_l^{D_2}) - \gamma \left[u'(\widetilde{c}_l^{D_2}) - u'(\widetilde{c}_l^{D_1}) \right]} \tag{71}$$

Similarly, combining (69) with (25), we can get

$$\frac{[1 - T_2'(\alpha_{2l} L_{2l})]}{(1 + T_{p_l})} = \frac{[1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)] \left[n_l - \gamma \frac{u'(\widetilde{c}_l^{D_2})}{u'(c_l^{D_2})} \right]}{[n_l - \gamma]} +$$

$$+ \pi_2(1+\lambda) + \frac{\pi_1(1+\lambda) \left(n_l u'(c_l^{D_2}) - \gamma \left[u'(\widetilde{c}_l^{D_2}) - u'(\widetilde{c}_l^{D_1}) \right] \right)}{n_l u'(c_l^{D_2})} \tag{72}$$

Equations (70), (71) and (72) define T_{p_l}, $T_1'(\alpha_{1l} L_{1l})$ and $T_2'(\alpha_{2l} L_{2l})$. It can be noticed that these marginal taxes can be chosen in several ways as long as the equations are satisfied. For instance, one of the three marginal taxes can be equal to zero as long as the other two are chosen optimally. These equations also show that taxing only the premium is generally not enough: this is only possible if $\widehat{L}_{2h} = \widehat{L}_{1h}$. Otherwise, in addition to the marginal tax on the premium, a marginal tax or subsidy is needed in at least one of the two dependence states.